Antonine Physics A2

Topic 12 Radioactivity and Nuclear Physics

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This is an electronic book which you can download. You can carry it in a portable drive and access it from your school's computers (if allowed) as well as your own at home.

This unit looks at the key discoveries that led to modern physics as we understand it and apply it.

Radioactivity was a key discovery in the late Nineteenth Century. As well as being an interesting phenomenon in its own right, it also enabled physicists to look at the atom in more detail. This in turn led to the ideas of nuclear physics

Nuclear Energy is a spin-off from nuclear physics. Although its application in weaponry is terrifying, the energy can be put to peaceful use in nuclear power stations. Although nuclear power remains highly controversial, the technology can meet our energy needs with very little carbon emission.

Topic 12	
1. Nuclear Physics	
Tutorial 12.01 α, β and γ Decay	
All Syllabi	
Contents	
12.011 Simple Models of the Atom	12.012 Isotope Notation
12.013 Unstable Nuclei	12.014 Safe Working with Radioisotopes
12.015 Background radiation	12.016 Production of Alpha Particles
12.017 Particles in Magnetic Fields	

12.011 Simple Models of the Atom (Revision)

It may be useful for you to revise what you did in AS level.

You will remember the simple model of the nuclear atom as proposed by Nils Bohr (*Figure 1*):

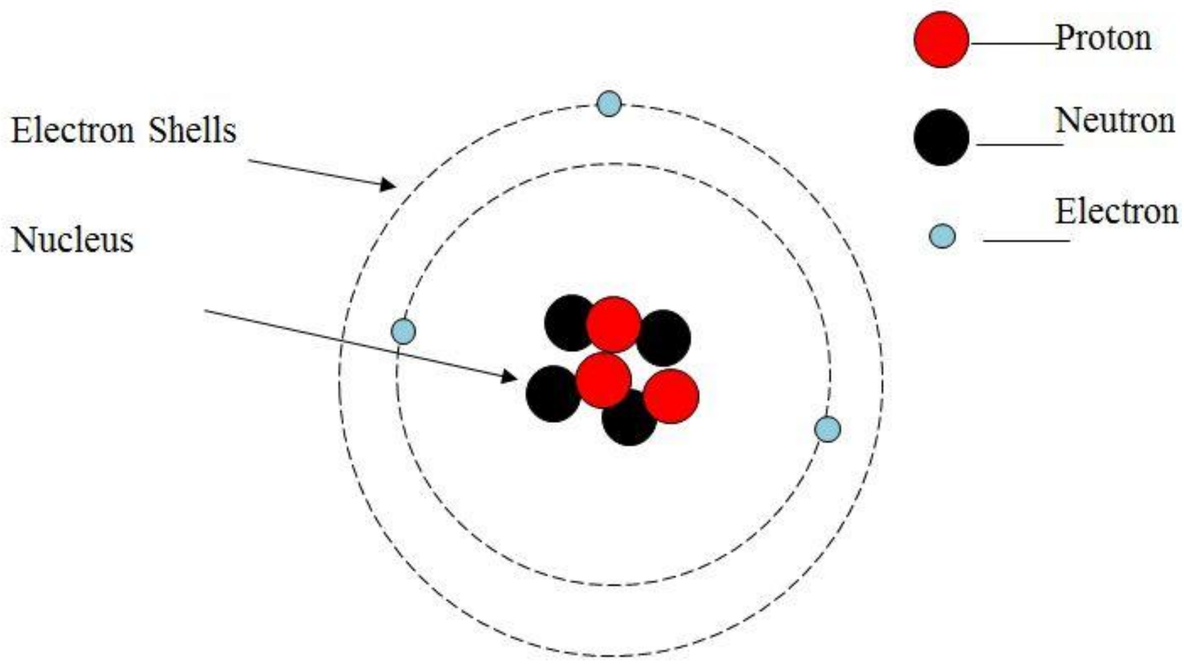


Figure 1 Simple atomic structure of a lithium atom

This is a **neutral** lithium atom. The number of protons is the same as the number of electrons. The electrons are in **shells**. The first shell can contain 2 electrons only. The next shell can contain up to 8 electrons.

12.012 Isotope Notation

Different atoms are distinguished by their numbers of protons and neutrons. We write the symbols using the following notation:

$$^{A}_{Z}\mathrm{X}$$

Figure 2 General isotope notation for a nuclide

- A is called the **nucleon** number, or the **mass** number. It is the total number of nucleons.
- Z is the **proton** number or the **atomic** number, which is the number of protons. The number of protons determines the element.

Therefore, lithium is written like this:

$$^{7}_{3}\mathrm{Li}$$

Figure 3 Isotope notation for a lithium atom

Be careful not to confuse atomic number with the symbol A. We will refer to A as the nucleon number in these notes and Z as the proton number.

We can determine the **number of neutrons** simply by subtracting the **proton number** from the **nucleon number**. (No of neutrons = $A - Z$). Atomic particles are always in whole numbers.

- **Isotopes** have the same numbers of protons, but different numbers of neutrons.
- Isotopes have **the same** chemical properties. Some physical properties are different, e.g. density.
- If the proton number is altered, the element changes.

Some isotopes are radioactive, as the nuclei are unstable.

Chemical reactions involve the electrons of the outer shells. Nuclei are not involved in any way and remain totally unaltered even in the fiercest chemical reactions.

12.013 Unstable Nuclei

Radiation is the process by which an unstable **parent** nucleus becomes more **stable** by **decay** into a **daughter** nucleus by emitting **particles** and/or **energy**. The basic form can be summed up as (*Figure 4*).

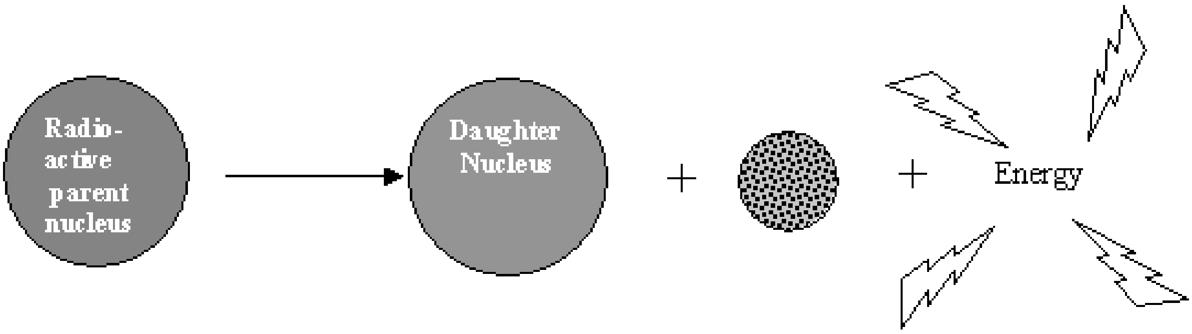


Figure 4 Transmutation

The decay can consist of several steps. The unstable nucleus can decay to another nucleus of a different atom by a process called **transmutation**. If the new nucleus is unstable, it will decay again. This is known as a **decay chain**. There may be several steps, some of which last a very long time indeed, or can be very short. Some elements have a decay time of thousands of millions of years. In others the decay time can be microseconds.

Elements have different **isotopes**. An element and its isotope have:

- The same number of protons (and electrons)
- Different numbers of neutrons.

If the isotope is unstable, it is **radioactive** and is called a **radioisotope**. We must be aware that radioactive decay is NOT the same as nuclear fission.

There are three kinds of radiation:

- **Alpha** (α) – a helium **nucleus**.
- **Beta** (β) – a high speed **electron**.
- **Gamma** (γ)– an **electromagnetic** radiation of wavelength about 10^{-14} m.

These kinds of radiation can be emitted individually or in any combination, depending on the type of isotope that is emitting the radiation. Often when an alpha particle is emitted the nucleus is **excited** and releases the excess energy in the form of a **gamma ray** or gamma photon.

When specimens of radioactive isotopes decay, they do so entirely **randomly**. There is no pattern whatsoever, and the rate of decay is not affected by temperature or other physical constraints, or chemical reactions.

The table helps us to compare the properties of radiation.

Radiation	***Description***	***Penetration***	***Ionisation***	***Effect of E or B field***
Alpha (α)	Helium nucleus $2p + 2n$ $Q = +2\,e$	Few cm air Thin paper	Intense, about 10^4 ion pairs per mm.	Slight deflection as a positive charge
Beta (β)	High speed electron $Q = -1\,e$	Few mm of aluminium	Less intense than a, about 10^2 ion pairs per mm.	Strong deflection in opposite direction to a.
Gamma (γ)	Very short wavelength em radiation	Several cm lead, couple of m of concrete	Weak interaction about 1 ion pair per mm.	No effect.

We will look at the mechanisms of production of alpha and beta radiations later.

We need to be aware that elements with unstable nuclei can be harmful to living organisms.

- Alpha particles are **intensely ionising**. The good news is that they are stopped by a few cm of air or by the **skin**. The bad news is that if you ingest an alpha emitter, the radiation quickly will macerate the DNA of living cells, such as the lining of the intestines or lungs. Then you are in serious trouble. The main fear from the fall-out of a nuclear catastrophe is from alpha emitters (although you wouldn't want to take a gamma source to bed with you).
- Beta particles can penetrate the body but are stopped by a few mm of Aluminium. They are less damaging than gamma rays or alpha particles. They are weakly ionising. **Medical tracers** are radioisotopes that are beta emitters
- Gamma rays are considered the most dangerous form of radiation, as they are very penetrating. They are **attenuated** (reduced) by several centimetres of lead but not stopped completely. So, they can pass easily through our bodies. Surprisingly, they cause very little **ionisation**, which causes genetic damage, and are not absorbed very efficiently by DNA, so quite a long exposure to gamma rays is needed to destroy DNA completely. However random damage can be done by smaller doses. It can be repaired by the cell's repair mechanisms, but mis repair can cause mutations, which can lead to cancer. Intense radiation can mess up DNA sufficiently to cause radiation sickness. This can of course apply to other radiations as well.

In the early days of radiation research, people had little clue as to how dangerous the stuff was. In those days lumps of uranium were used as icebreakers at parties ("*Darling, do come and feel my magic metal*."); the metal felt warm, and gave the person feeling it a massive dose of radiation! Today the nuclear industry takes safety very seriously indeed, and workers are rigorously monitored. If it appears that personnel are being exposed to higher levels of radiation than they should be, they are withdrawn from that work. Safety must the primary consideration in every function of the nuclear industry. However, things can go wrong as in any human activity, e.g. falsification of records, or unauthorised experiments, such as those that led to the Chernobyl disaster, when 7 tonnes of caesium-137 was scattered over Europe.

12.014 Safe Working with Radioisotopes

Those who work with radiation are issued with a **film badge**, as shown in the picture below (*Figure 5*).

Figure 5 A film badge

The film is exposed to all kinds of radiation. There are strips of metal that stop alpha, beta, and lead that attenuates gamma. Every month the film is taken in and processed, and a new film is issued. If the workers are found to be exposed to a higher than safe level of radiation, they have to be removed from that line of work for a period of time. This is a rare event, because in reality most workers are exposed to little above background radiation anyway.

Those who handle radioactive materials are highly trained and aware of the risks. They will take elementary precautions such as:

- wearing a lead apron.
- carrying sources in lead-lined containers.
- handling sources with tongs and gloves.
- keeping the sources well away from themselves.
- not pointing sources at others.
- monitoring radiation levels with a portable Geiger counter.

Fixed sources of radiation are contained in cells or bunkers with thick concrete walls. The cells have interlocking to prevent access when the source is exposed. The interlocks will not unlock unless the source has been retracted.

12.015 Background radiation

Whenever radiation experiments are carried out, it is important to realise that there is always a certain amount of background radiation. Many elements have radioactive isotopes as well as stable isotopes. These will give off radiation.

While it's not important to know the sources of the background radiation, we must correct the count by subtracting the background radiation. Because the emission of backgrounds counts is a random process, it is not appropriate to take a momentary sample. To reduce uncertainty, we need to take a count of at least 60 seconds and divide it by 60 to get an average background count.

Corrected count (Bq) = Total count (Bq) - background count (Bq) .... *Equation 1*

Background radiation is entirely normal, and we are adapted to cope with it.

12.016 Production of Alpha Particles

Alpha particles come from **heavy** elements of mass greater than 106 atomic mass units. In classical physics, the strong force balances the electro-magnetic force, so the alpha does not have the energy to get out.

In quantum physics, there is a small chance that the alpha can get out by a process of **quantum tunnelling** (*Figure 6*).

At this level we assume that all the energy is kinetic. Alpha particle energy is between 3 and 7 MeV.

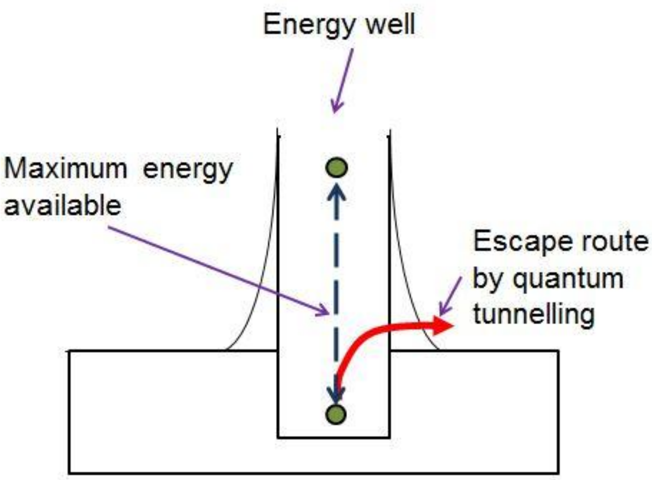


Figure 6 Quantum tunnelling

12.017 Particles in Magnetic Fields

You may want to revise the motion of charged particles in magnetic fields in Topic 11. If the particles are deflected by a magnetic field going into the screen, the result is this (*Figure 7*).

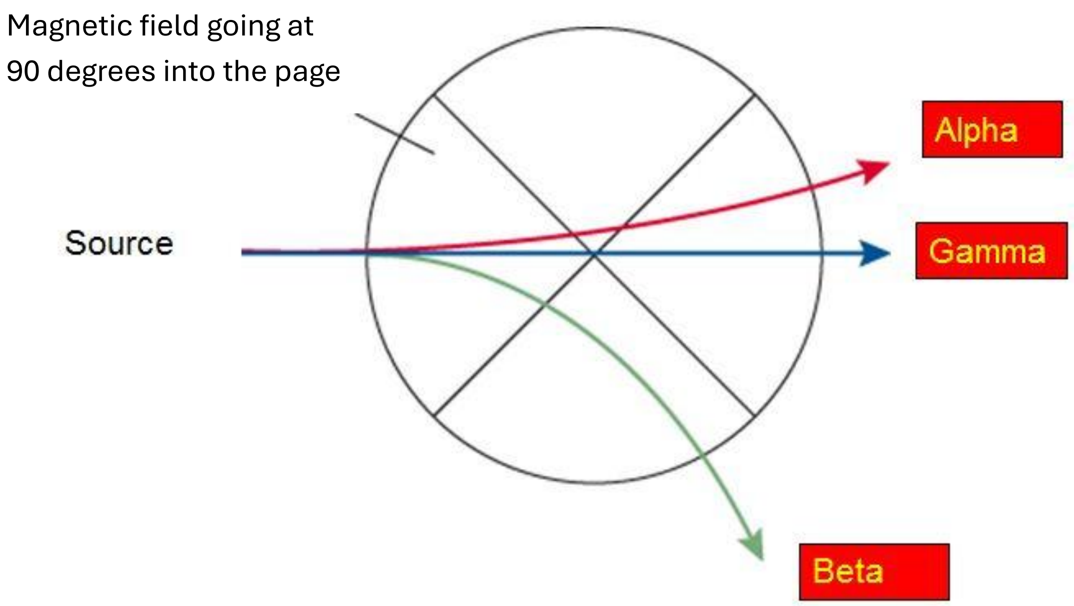


Figure 7 Effect of a magnetic field on emitted radiation

The deflections tell us not only the charge on the particle, but the mass and the speed of the particles. The equation derived in Magnetic Fields is applicable:

$$v = \frac{BQr}{m}$$

.... *Equation 2*

If we know the kinetic energy of a particle, we can then work out the velocity. Alpha particles have kinetic energy between 3 and 7 MeV.

When using Fleming's Left Hand Rule to determine the movement of a charged particle, remember that the current is regarded as **conventional** current. Electrons move in the **opposite** direction to conventional current.

Tutorial 12.01 Questions

12.01.1

What is meant by the term transmutation?

12.01.2

Complete the table that describes the properties of the three common radiations.

Radiation	*Particle*	*Range in air*	*Stopped by*
Alpha			
Beta			
Gamma			

12.01.3

Explain the dangers associated with radioactive sources.

12.01.4

Alpha and beta particles lose about 5×10^{-18} J of kinetic energy in each collision they make with an air molecule. An alpha particle makes about 10^5 collisions per cm with air molecules, while a beta particle makes about 10^3 collisions. What is the range of an alpha particle and a beta particle if both particles start off with an energy of 4.8×10^{-13} J?

12.01.5

An alpha particle has a kinetic energy of 5.45 MeV. It is travelling horizontally across the screen from left to right. It is deflected by a magnetic field of 0.86 T which is perpendicular to the screen and going into the screen.

(a) Calculate the speed of the particle.

(b) Calculate the radius of curvature. Give your answer to an appropriate number of significant figures.

(c) In which direction, upwards or downwards, is it deflected?

Mass of an alpha particle = 6.64×10^{-27} kg. Electronic charge = 1.60×10^{-19} C.

Tutorial 12.02 Evidence for the Nucleus

All Syllabi

Contents

12.021 Rutherford Scattering

In the early part of the last century, the accepted model of the atom was proposed by J J Thompson in his **plum pudding** model. This consisted of a matrix of protons in which were embedded electrons.

Ernest Rutherford (1871 – 1937) used **alpha particles** in a vacuum to study the nature of atomic structure with the following apparatus (*Figure 8*).

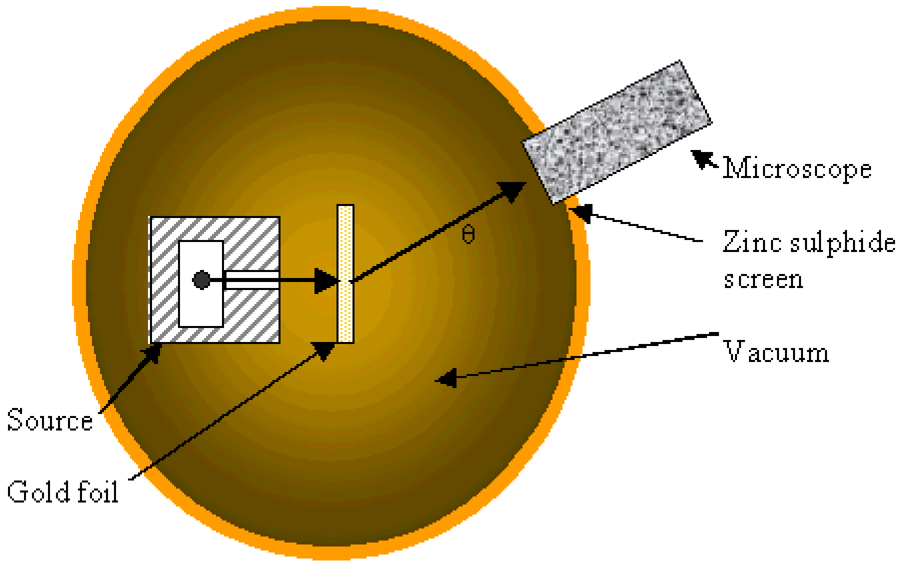


Figure 8 Rutherford' alpha scattering experiment

Rutherford was using alpha particles (**helium nuclei**) as nuclear bullets to smash up the atoms; he wanted to see atoms bursting like watermelons. But...

His observations are best illustrated with this diagram (*Figure 9*)

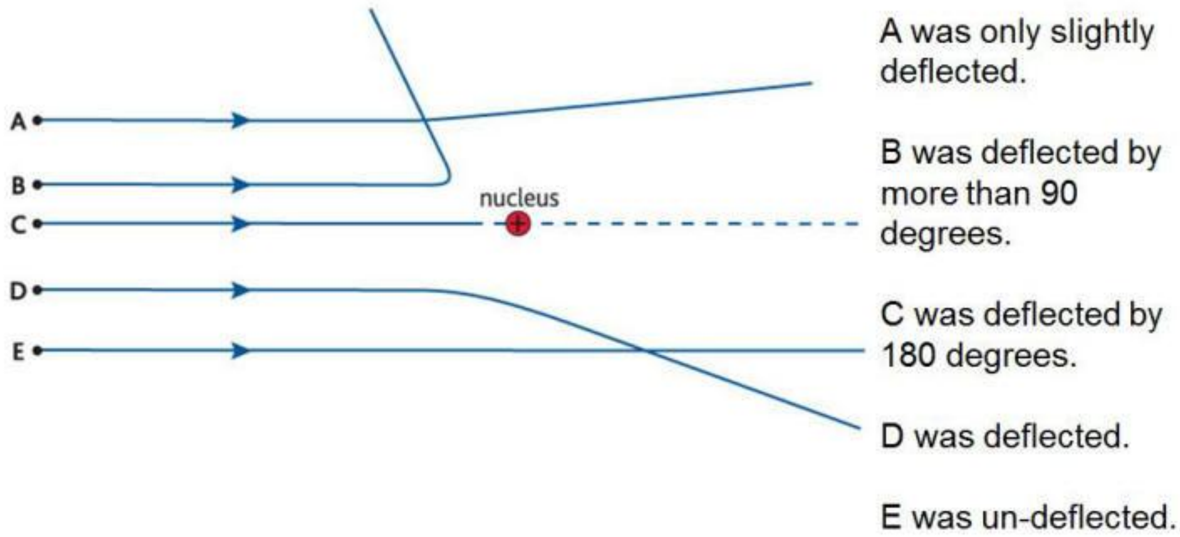


Figure 9 Results of Rutherford's alpha scattering experiment

Instead of bits of atom, Rutherford found that a small proportion of the alpha particles were deflected, while an even smaller proportion bounced right back. From analysis of these observations, he concluded:

- Most of the atom was empty space.
- The positive charge was concentrated in a very small space
- The radius of the nucleus was in the order of 3×10^{-14} m.
- The alpha particles that were deflected back had to be travelling in a line with the nucleus.

Rutherford's estimates were not far out. Later research has shown the nuclear radius to be in the order of 1.5×10^{-14} m. However, the boundary is not sharp, but rather fuzzy, as the nucleus is a very dynamic entity.

We need to remember that most of the volume of an atom is empty space. The general size of an atom is about 10^{-10} m (0.1 nm) and that is determined by the electron clouds. The size does not vary much across the elements. A calcium atom is about the same size as a gold atom. Even the heavy gold nucleus has a tiny diameter compared to the diameter of the atom.

12.022 Limitations

There are **limitations** to Rutherford's calculations:

- The distance is the distance of the closest approach.
- This is determined by the energy of the alpha particle.
- The higher the energy, the closer it will get.

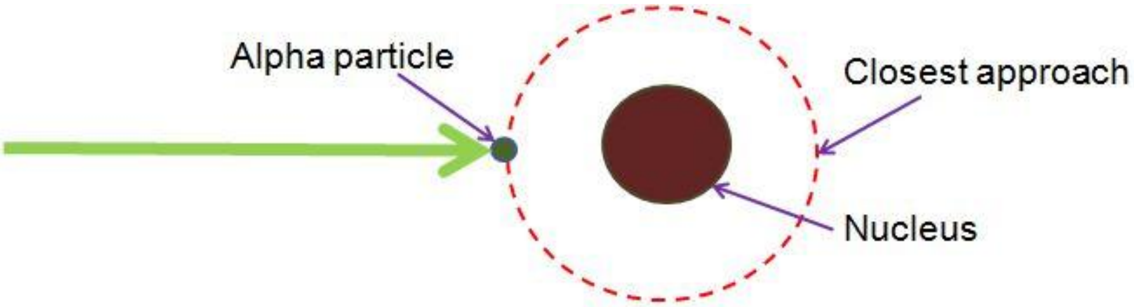


Figure 10 Closest approach of an alpha particle to a nucleus

We can use electrical potential energy to get an estimate of the closest approach. From Electric Fields, we saw:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r}$$

.... *Equation 3*

In the calculation you are going to do, the alpha particle will have an energy of 5 MeV. We can assume that it is kinetic energy. In reality, there is a certain amount of vibrational energy, but this is small compared to the kinetic energy. The proton number for gold is 79 and the proton number for the alpha particle is 2. (You knew that, didn't you?)

The nuclear radius is actually rather smaller than this, about 1×10^{-14} m. Most atoms have a radius of about 1×10^{-10} m, and that is remarkably consistent between atoms. This is because the mass is contained in the nucleus, which is 10 000 times smaller. As each electron has a mass of 1/1800 the mass of a proton, the mass contributed by the electrons in their shells is very small.

12.013 Rutherford Scattering at High Energy

If the alpha particles have a very high energy, their behaviour deviates from what is predicted by the Rutherford Equation (not on the syllabus). The deflection angle in this case is set at 60°, rather than being varied. The idea is shown below (*Figure 11*).

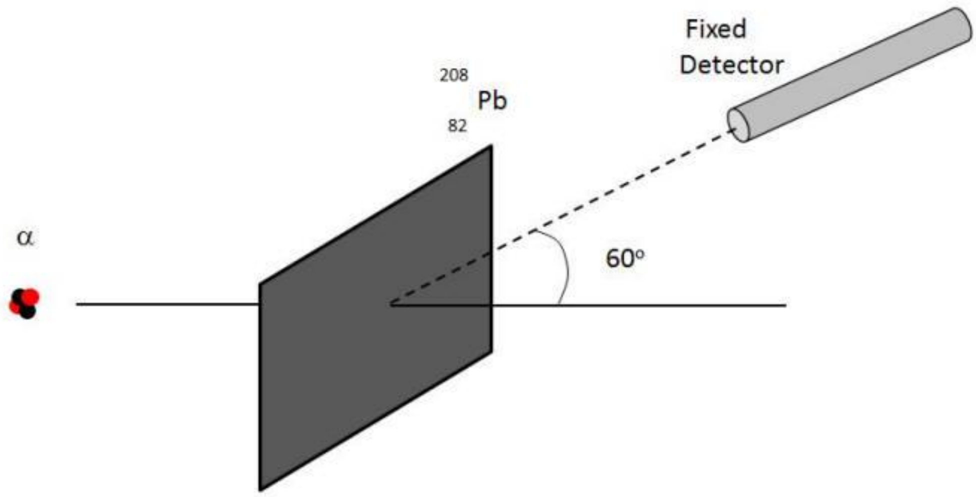


Figure 11 Alpha particles being detected at a fixed angle

Alpha particles of different energies are used. This can be achieved by **accelerating** the alpha particles using an electric field. The only alpha particles measured are those that are deflected by 60°. The relative intensity of the deflected alpha particles is measured by the detector. The data are recorded and are shown on the graph below (*Figure 12*).

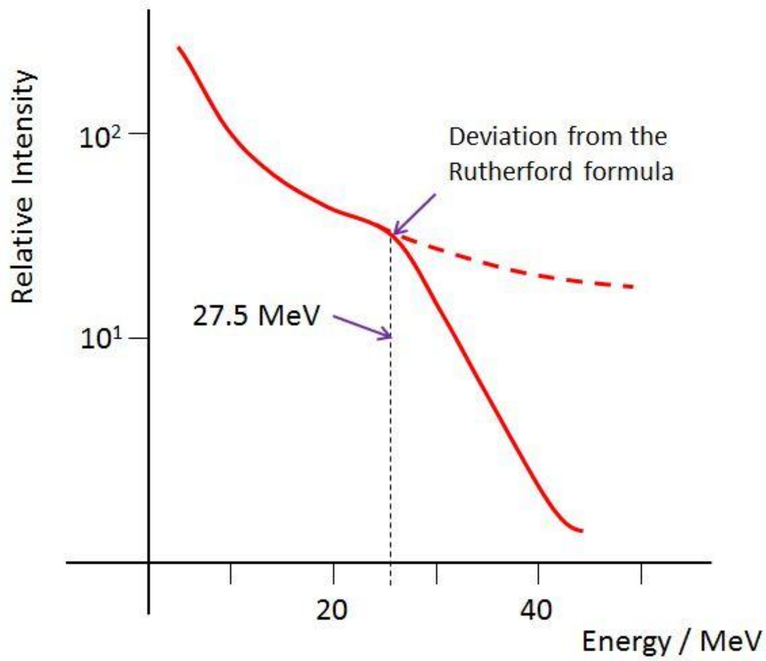


Figure 12 The response of increased energy of alpha particles compared with the prediction

The Rutherford Equation predicts the number of alpha particles per unit area striking the detector is inversely proportional to the square of the kinetic energy, i.e.:

$$N \propto \frac{1}{{E_k}^2}$$

.... *Equation 4*

This graph shows that the model is followed until the alpha particles have an energy of about 27.5 MeV. Then the relative intensity falls much more rapidly that what is suggested by the model. The dotted line shows the expected results if the Rutherford Equation were followed.

The conclusion from high energy alpha particle experiments was that the nucleus had a radius of about 7.1 fm (7.1×10^{-15} m).

Tutorial 12.02 Questions

12.02.1

What led Rutherford to conclude that the nucleus was very tiny and had a positive charge?

12.02.2

A 5 MeV alpha particle approaches a gold nucleus.

(a) What is the charge carried by the alpha particle?

(b) What is the charge carried by the gold nucleus?

(c) What is the energy in joules of the alpha particle?

(d) What is the minimum approach distance?

Tutorial 12.03 Nuclear Instability	
All Syllabi	
Contents	
12.031 Stability of Nuclei	12.032 Alpha Decay
12.033 Beta Minus Decay	12.034 Beta Plus Decay
12.035 Decay Chains	12.036 Excited Nuclei
12.037 Energy levels in Nuclei	12.038 Metastable states

12.031 Stability of Nuclei

The chemical properties of any element are governed by the number of protons, the **proton number**, which is given the code Z. The **stability** of the nucleus depends on a combination of the proton number and the neutron number. We can plot a graph of the number of neutrons (given by the difference between the mass number and the proton number) against the proton number. The general pattern is like this (*Figure 13*).

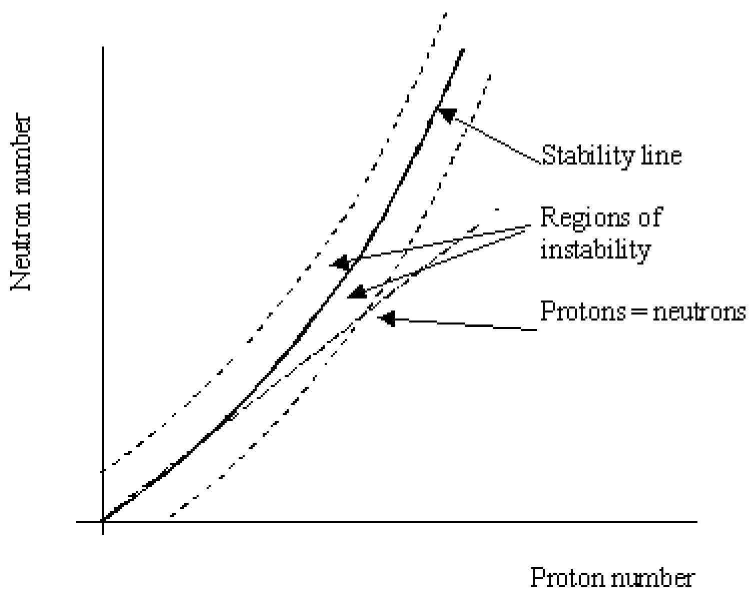


Figure 13 Showing the stability of nuclides

The stability line is sometimes called the **valley of stability**. It is not a straight line where the number of protons = number of neutrons. Lithium has three protons and four neutrons, for example. While lithium 6 is stable, it is only 8 % of the total lithium found in nature. 92 % is lithium 7.

This more detailed image (*Figure 14*) has neutron number on the horizontal axis and the proton number on the vertical axis. Make sure that you make it clear on your own sketch graphs.

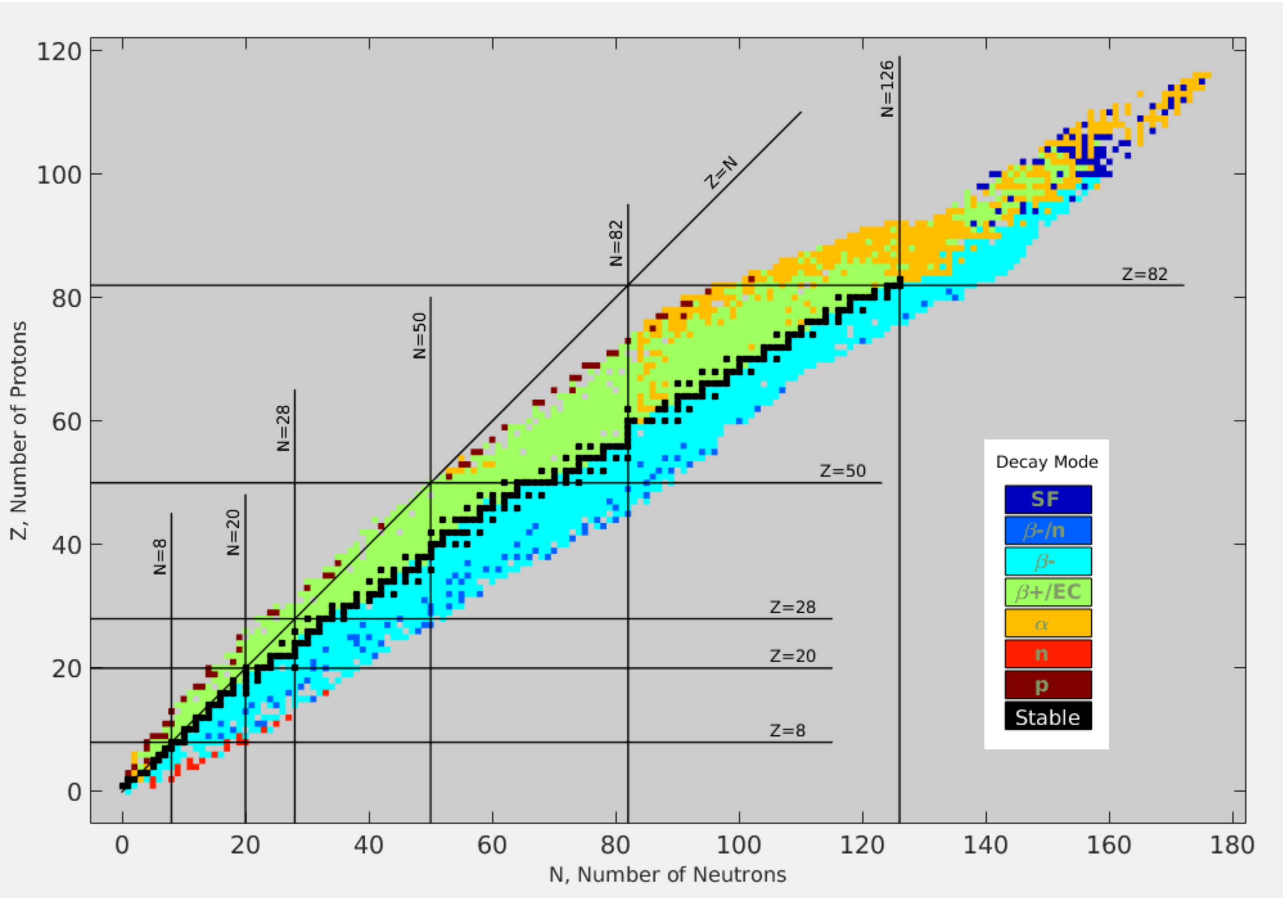


Figure 14 Plot of Z against N for nuclides (Image by Bdushaw - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=61302798)

For **stable nuclides**, we notice the following:

- The lightest nuclides have almost equal numbers of protons and neutrons.
- The heavier nuclides require up to 50 % more neutrons than protons. The greater number of neutrons is needed to stop the nucleus flying apart, in effect diluting the repulsive force of the positively charged protons.
- Most nuclides have both an even number of protons and an even number of neutrons.
- Alpha particles are made of two protons and two neutrons. Certain elements like silicon, oxygen, and iron have a similar ratio of protons and neutrons.

For **unstable nuclides**, we see:

- Disintegrations tend to produce new nuclides that are nearer the stability line and carry on until the stability line is reached.
- Nuclides above the line decay so that the proton number increases by 1, i.e. a beta emission.
- Nuclides below the line decay to reduce the proton number and the proton to neutron ratio increases. This is achieved by alpha decay.

Beta plus decay also occurs where the nucleus is beneath the line of stability. In this case a proton turns into a neutron and a **positron** (positively charged anti-electron) is given off.

12.032 Common Modes of Decay - Alpha

Alpha radiation mostly comes from heavy nuclides with proton numbers greater than 82, but smaller nuclides deficient in neutrons can also be alpha emitters. It is believed that the alpha particle is formed some time before its emission, and it gains its energy from the **mass defect** in the nucleus. The term Q stands for the **energy**. The general decay equation is summarised below.

$$^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He} + Q$$

...............*Equation 5*

We should note the following:

- The alpha particle is a helium **nucleus** (NOT atom)
- Energy is released in the decay. The quantity is precise, according to the nuclide. The energy is **kinetic**, with the majority going to the alpha particle and a little going to the decayed nucleus. Some nuclides emit all their alpha particles at one energy, while others emit them at two or more discrete energy levels.
- The velocity of the alpha particle is much greater than that of the nucleus.
- The **nucleon number** goes down by 4, the **proton number** by 2.
- A typical alpha decay is:

$$^{228}_{90}\mathrm{Th} \longrightarrow {}^{224}_{88}\mathrm{Ra} + {}^{4}_{2}\mathrm{He} + Q$$

.... *Equation 6*

Alpha particles are **intensely ionising**. They smash through air molecules, knocking off electrons as they go. However this reduces the kinetic energy, so that in the end they stop. Then they pick up a couple of free electrons to become helium atoms. To collect an appreciable sample of helium from an alpha emitter would take a very long time.

12.033 Common Modes of Decay - Beta Minus

Neutron rich nuclei tend to decay by **beta minus (β–) emission**. The beta particle is a **high-speed electron** ejected from the nucleus, NOT the electron clouds. It is formed by the decay of neutrons, which are slightly more energetic than a proton. Isolated protons are stable; isolated neutrons last about 10 minutes.

The neutron, having emitted an electron, is converted to a proton, and this results in the proton number of the nuclide going up by 1. A new element is formed. The reaction at the nucleon level is:

$$^{1}_{0}\mathbf{n} \longrightarrow {}^{1}_{1}\mathbf{p} + {}^{0}_{-1}\mathbf{e} + {}^{0}_{0}\bar{\nu}_e + \mathbf{Q}$$

.... *Equation 7*

Notice that as well as the neutron (n) and the proton (p), the beta particle is represented as an electron (e). The strange symbol:

$$^{0}_{0}\bar{\nu}_e$$

(pronounced 'noo-bar e') is a strange little particle called an **electron antineutrino**. The general equation for β- decay is:

$$^{A}_{X}\mathbf{X} \longrightarrow {}^{A}_{Z+1}\mathbf{Y} + {}^{0}_{-1}\mathbf{e} + {}^{0}_{0}\bar{\nu}_e + \mathbf{Q}$$

.... *Equation 8*

A typical decay is:

$$ {}^{29}_{13}\mathrm{Al} \longrightarrow {}^{29}_{14}\mathrm{Si} + {}^{0}_{-1}\mathrm{e} + {}^{0}_{0}\bar{\nu}_e + Q $$

.... *Equation 9*

Notice that:

- The nucleon number remains the same.
- The proton number goes up by 1.
- The beta particle is created at the instant of the decay.
- The antineutrino is very highly penetrating and has a tiny mass. It is very hard to detect, as it rarely interacts with matter.
- The antineutrino has to be present to ensure that **momentum is conserved**.
- A precise amount of energy is released, according to the nuclide.
- That energy is shared among the nucleus, the electron and the antineutrino.

The **neutrino** was first proposed by Wolfgang Pauli (1900 - 1958) in about 1930 to explain how energy and momentum could be conserved in a beta minus decay. At that time the neutron was not yet discovered. (This was done in 1932 by James Chadwick (1891 - 1974).) The term neutrino ("neutral little thing") was coined by Enrico Fermi (1901 - 1954). Evidence was revealed by the way that the proton that was formed in the beta decay recoiled in a slightly different direction to that expected. The idea is shown below in *Figure 15*.

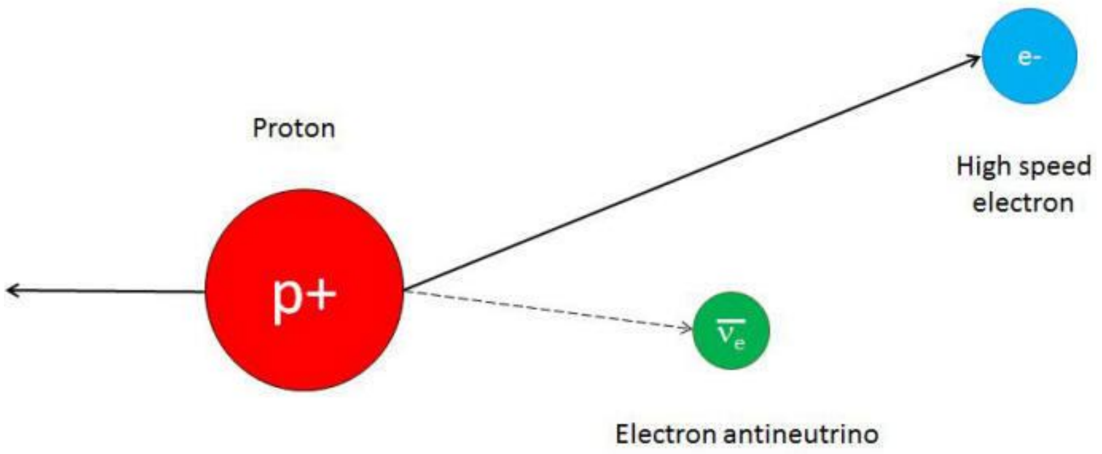


Figure 15 Beta minus emission

Conservation of momentum rules suggested that there must have been a third very tiny particle, which he called the **neutrino** (neutral little thing). Later it was called the **electron neutrino** (as it was associated with an electron). However, the use of **quantum numbers** showed that it must be an **electron anti-neutrino**.

The proportion of shared energy is variable, so there is a range of energies of the b- particles. The graph (*Figure* 16) shows a typical distribution.

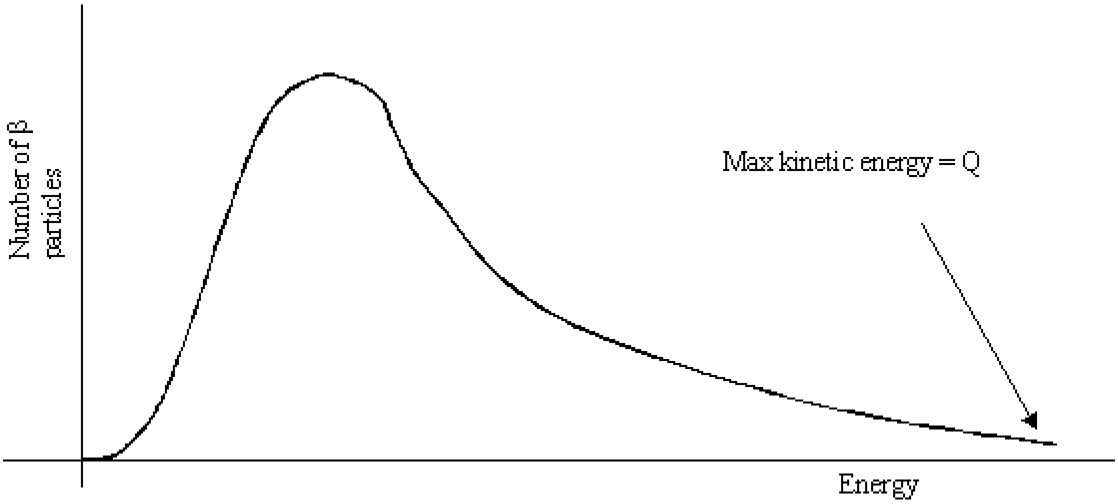


Figure 16 Distribution of Energies of beta minus particles

If beta particles are emitted in a medium where the speed of light is lower than that of the ejected electrons, then the passage of the electron is accompanied by an optical shock wave, like the sonic boom of a supersonic aeroplane. The resulting glow is called **Cherenkov radiation**.

12.034 Beta Plus Decay

The **positron** is the **anti-particle** to the electron. It has the same size, but opposite charge. Beta-plus (β+) decay involves the emission of a positron. It rarely occurs naturally and is generally found in nuclear physics experiments in reactors. If we bombard fluorine atoms with alpha particles, we get a radioisotope of sodium, which decays by positron emission.

$$^{19}_{9}\mathrm{F} + ^{4}_{2}\alpha \longrightarrow ^{22}_{11}\mathrm{Na} + ^{1}_{0}\mathrm{n}$$

.... *Equation 10*

The second reaction is:

$$ {}^{22}_{11}\mathrm{Na} \longrightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{+1}\overline{\mathrm{e}} + {}^{0}_{0}\nu_e $$

.... *Equation 11*

Here we see a **positively charged electron**, the **positron** being emitted with an **electron neutrino** (n_e). At the nucleon level we see:

$$ {}^{1}_{1}\mathrm{p} \longrightarrow {}^{1}_{0}\mathrm{n} + {}^{0}_{+1}\overline{\mathrm{e}} + {}^{0}_{0}\nu_e $$

.... *Equation 12*

The proton is turned into a neutron.

There is another way that a proton is turned into a neutron, and that is by **electron capture**. An electron is captured from the electron cloud. As another electron falls to take over the vacancy left, an X ray is emitted. The general scheme is:

$$ {}^{A}_{X}\mathrm{X} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{A}_{Z-1}\mathrm{Y} + {}^{0}_{0}\nu_e $$

.... *Equation 13*

And at the nucleon level we see:

$$ {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\mathrm{e} \longrightarrow {}^{1}_{0}\mathrm{n} + {}^{0}_{0}\nu_e $$

.... *Equation 14*

Note that prior to the emissions, the electrons, positrons, neutrinos, or antineutrinos do not exist as separate entities within the nucleus. They are created at the instant of the decay. Free neutrons outside the nucleus decay to protons by β- emission.

12.035 Decay Chains

When radioisotopes decay, there may be several steps before the nucleus achieves stability. We call this series of decays a **radioactive series** or a **decay chain**. There are different half-lives at each step, some of which can be extremely long, while others are short. We can represent these graphically as shown below (*Figure 17*).

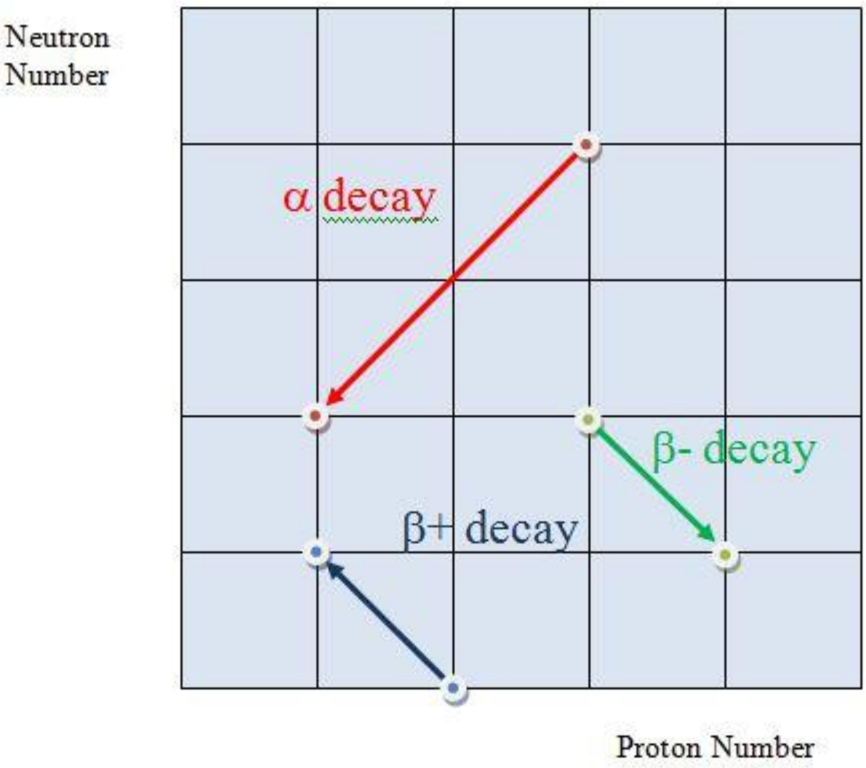


Figure 17 Moves in a decay chain

There are different permitted moves, according to the decays involved. Here is a **decay chain**.

$$^{232}_{90}Th \xrightarrow{\alpha} {}^{228}_{88}Ra \xrightarrow{\beta-} {}^{228}_{89}Ac \xrightarrow{\beta-} {}^{228}_{90}Th \xrightarrow{\alpha} {}^{224}_{88}Ra \xrightarrow{\alpha} {}^{220}_{86}Rn \xrightarrow{\alpha} {}^{216}_{84}Po$$

We can show this series graphically (*Figure 18*).

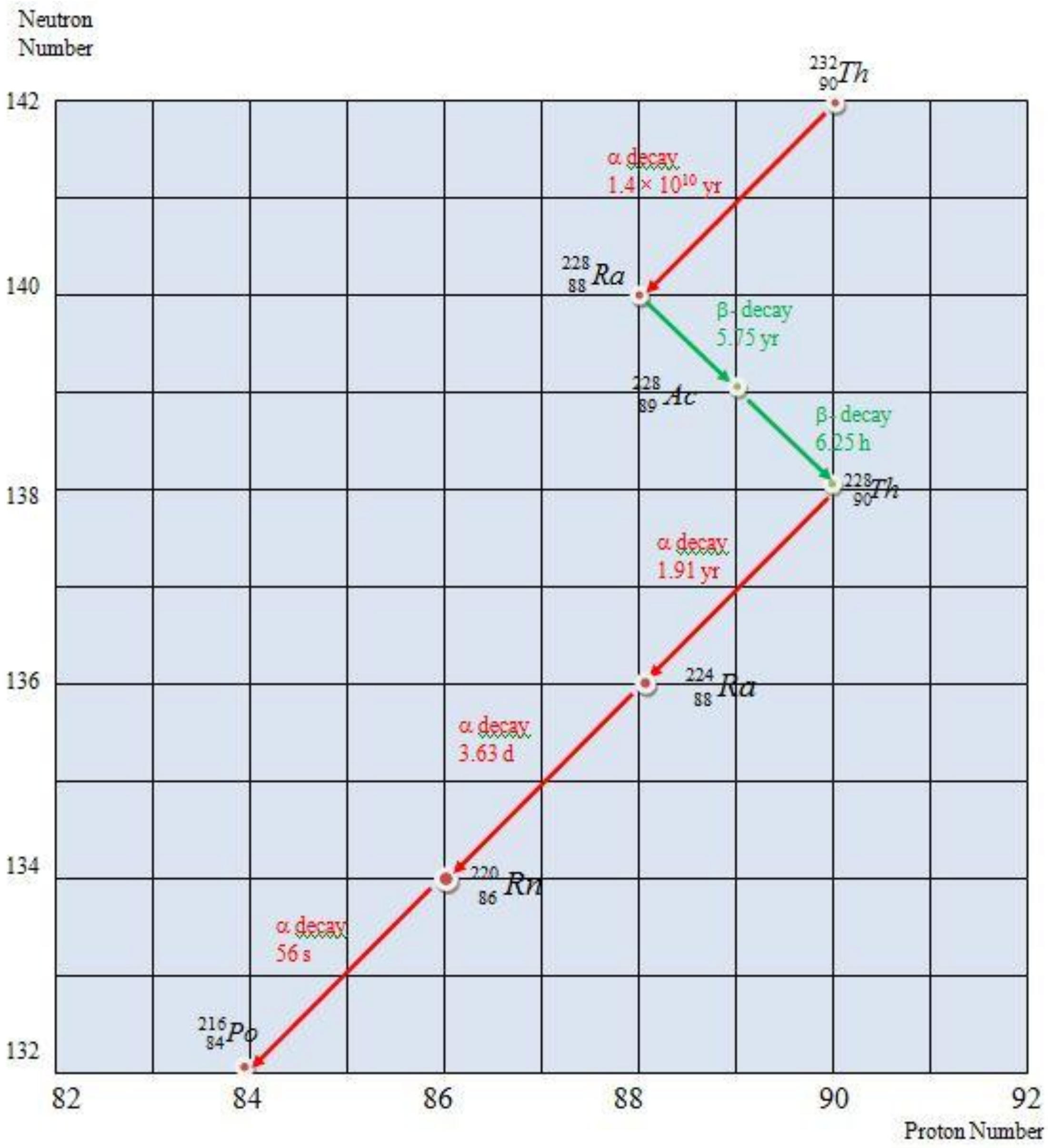


Figure 18 A typical decay chain

Remember that the neutron number is not the same as the nucleon number.

Neutron number = nucleon number – proton number

Note also that the emitted particle is NOT radioactive in itself.

12.036 Excited Nuclei

After alpha or beta decay, the daughter nucleus is often left in a very energetic state. We call that state **excited**. The nucleus gets rid of this energy in the form of a **photon** of **electromagnetic radiation** of very short wavelength, called a **gamma ray** (g-ray). Gamma rays, cosmic rays, and hard X-rays have the same frequency, so are really the same thing. Since photons are not particles, there is no change in the proton number, or the nucleon number. The nucleus becomes less energetic.

Some points to note:

- The nucleus is unaltered physically.
- The radiation is about the same size of the nucleus, about 10^{-14} m.
- The precise wavelength is a property of the nuclide involved.
- Gamma radiation causes little ionisation, so it's very penetrating.
- Gamma rays are created at the instant of the decay.

The energy comes from the mass defect. At the nuclear level the key idea is that mass and energy are interchangeable. There is a measurable change in mass of a nuclide emitting gamma rays over a long period.

Gamma rays have two important medical applications:

- **Radiotherapy** – a cobalt 60 source is aimed at a cancerous tumour. The genetic material of cancers is generally unstable, and the gamma ray photons can have sufficient interaction to render the cancerous cells unviable. Unfortunately, it can have the same effect on normal cells as well, and there are nasty side effects.
- Tracers such as technetium-99 can be injected and used to monitor blood flow using a **gamma camera**. This is an important diagnostic tool.

12.037 Energy levels in Nuclei

A nuclear event can be:

- an alpha decay.
- a beta minus decay.
- a beta plus decay.
- an electron capture event.

In any of these events, the daughter nucleus can be **energetic**. It can lose the excess energy by emitting a **gamma photon**. It then falls from the excited state to the ground state.

Consider this beta minus decay:

$$^{27}_{12}Mg \rightarrow ^{27}_{13}Al + ^{0}_{-1}e- + \overline{\nu_e}$$

.... *Equation 15*

The aluminium nucleus is excited, and can be shown in an **energy level diagram** (*Figure 19*)

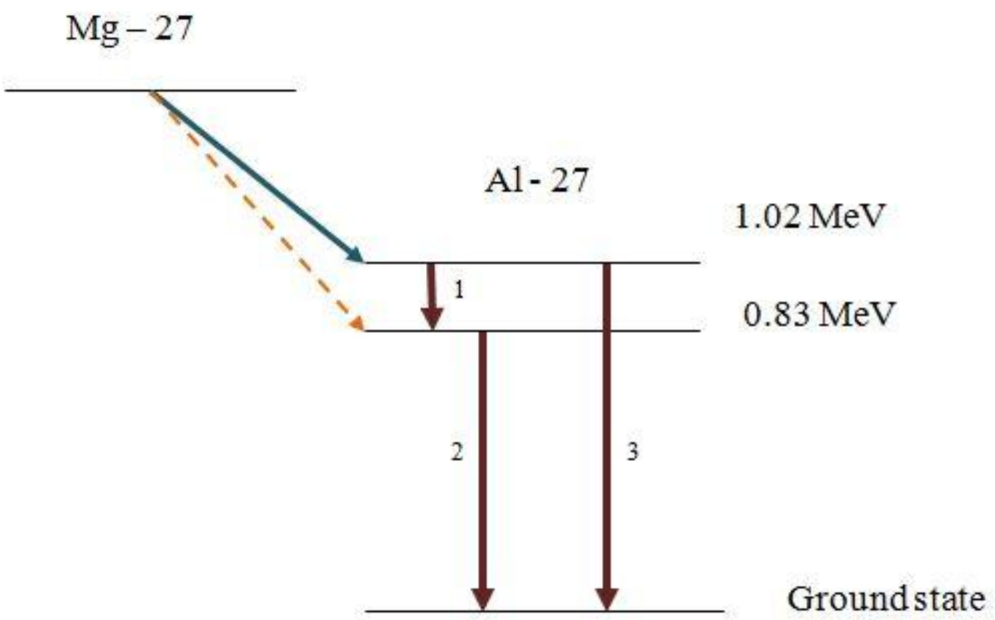


Figure 19 Energy level diagram for an excited aluminium-27 nucleus

In this particular decay, there are three possible energies for the gamma photon. These are shown in the diagram as transitions.

- Transition 1 is from 1.02 MeV to 0.83 MeV.
- Transition 2 is from 0.83 MeV to 0 (the ground state).
- Transition 3 is from 1.02 MeV to 0.

We can work out the energy of the gamma photon in joules by multiplying the energy by 1.6×10^{-19}. Then we could work out the frequency of the photon by using the equation:

$$E = hf$$ .... *Equation 16*

12.038 Metastable states

It is possible for a daughter nucleus to remain in an excited state for some time. One example is the element **technetium**, formed by beta decay from a radioactive decay from an isotope of molybdenum.

$$^{99}_{42}Mo \xrightarrow{\beta-} {}^{99}_{43}Tc$$ .... *Equation 17*

This can be shown in an energy level diagram (*Figure 20*).

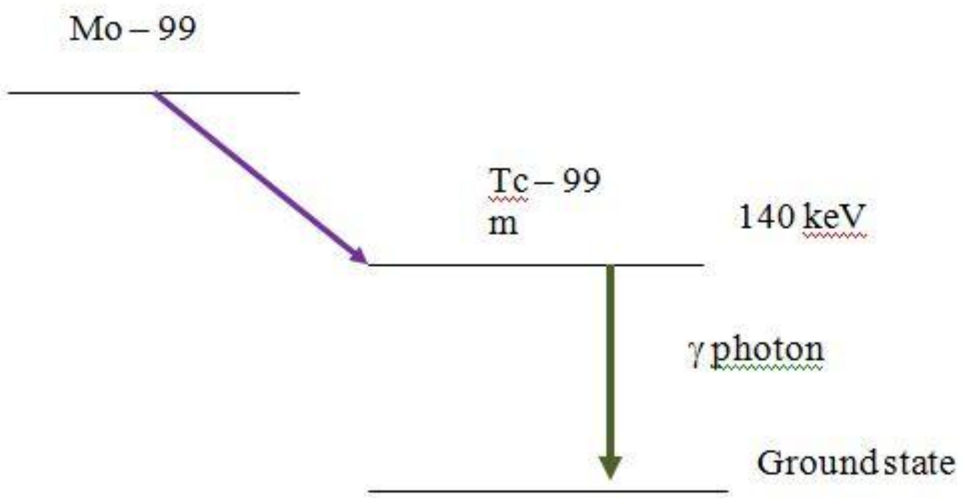


Figure 20 Energy level diagram showing metastability

We refer to the prolonged excited state as **metastable**.

The technetium drops to ground state by emitting a **gamma photon** of 140 keV. This is low enough to be much safer than other gamma sources. The half-life of the gamma emission is about 6 hours. Like all radioactive decays, the emission of gamma photons is random.

Technetium in its ground state decays by **beta minus emission** to ruthenium, with a half-life of 211 000 years. This adds very little additional radiation burden on the body. Most of it is excreted in the urine.

Other nuclides can be metastable, e.g. Mn-46m; Ar-32m; Zn-69m

Tutorial 12.03 Questions

12.03.1

$$^{228}_{90}\text{Th} \longrightarrow \, ^{224}_{88}\text{Ra} + \, ^{4}_{2}\text{He} + Q$$

Is this equation balanced? Explain your answer.

12.03.2

What is the balanced nuclear equation for the following decays?

(a) emission of a beta- particle from oxygen 19

(b) emission of an alpha particle from polonium 212

(c) emission of a beta + particle from cobalt 56

Proton numbers O – 8, F – 9, Fe – 26, Co – 27, Pb – 82, Po – 84

12.03.3

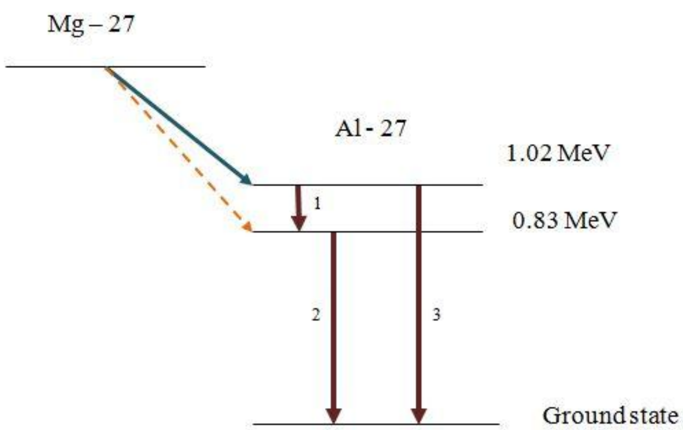
$$^{1}_{1}\text{p} \longrightarrow \, ^{1}_{0}\text{n} + \, ^{0}_{+1}\text{e} + \, ^{0}_{0}\nu_e$$

Is the charge conserved in this equation?

12.03.4

Explain how gamma rays are formed.

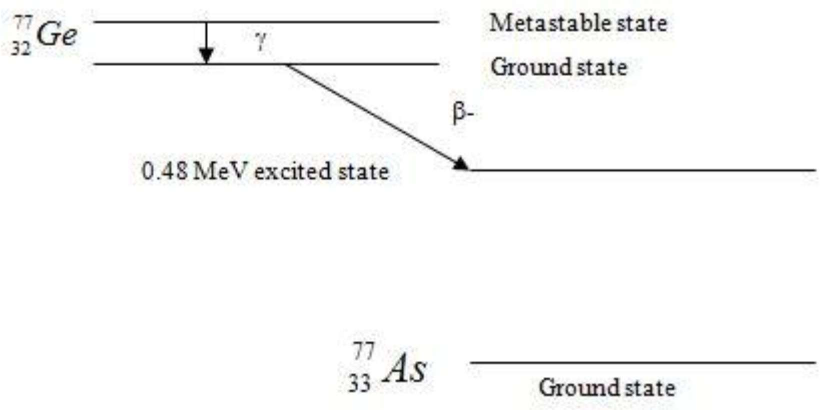
12.03.5



Calculate the energy change in transition 1. Express your answer in MeV and joules. Hence work out the wavelength of the gamma photon.

12.03.6

The germanium isotope Germanium 77 has a metastable state which decays to the ground state by emission of a 0.16 MeV gamma photon. The isotope decays by beta minus emission to form an arsenic isotope which is in an excited state 0.48 MeV above the ground state. This is shown in the diagram below:



(a) Complete the diagram to show the changes.

(b) Calculate the wavelength of the photon released by the metastable germanium nucleus.

(c) The excited state of arsenic 77 also decays to the ground state via an excited state which is 0.27 MeV above the ground state. Calculate the energies of the gamma photons emitted in this decay.

Tutorial 12.04 Inverse Square Law of Gamma Radiation

All Syllabi

Contents

12.041 Measuring Radiation

To measure how the intensity of gamma radiation varies with distance, we can set up the following apparatus (*Figure 21*).

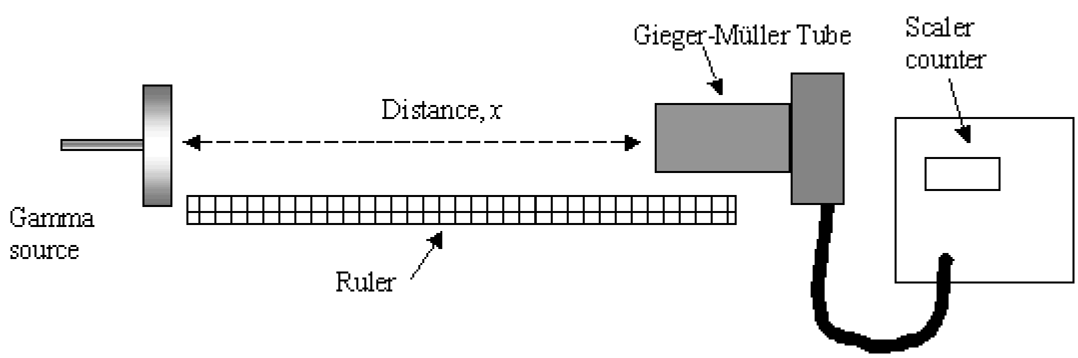


Figure 21 Measuring the intensity for gamma radiation

We carry the experiment out like this:

- Place the GM tube near the source.
- Count for 1 minute, and divide by 60 to get the count rate per second.
- Move away in steps of 5 cm until very low count rates are obtained.
- Use increased time samples for low rates to get an accurate count rate per second.
- Find out the background count rate.
- Correct the count rate using the background count rate.

If we plot distance ^{2} (y-axis) against 1/Count Rate, we will get a straight-line graph (*Figure 22*).

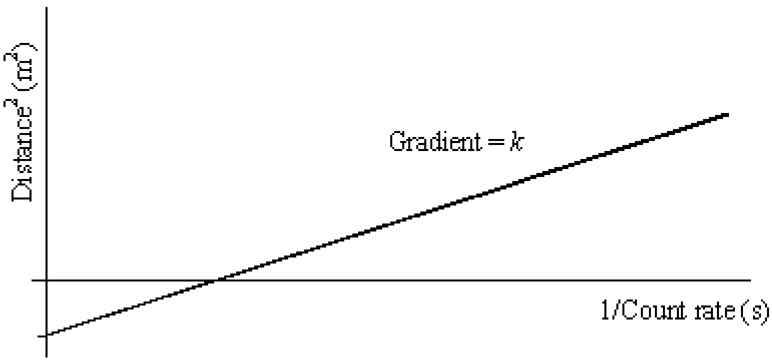


Figure 22 Plot of square of the distance against 1/Count rate

Notice that the line makes an intercept with the y-axis below the origin. This is because the gamma source is deep within its container. It would clearly be most undesirable to have it exposed immediately to the room. The intercept gives us the count rate right at the source.

This is a **required practical** (Number 12).

12.042 Inverse Square Law

Gamma rays obey the **inverse square law**, as they are an electromagnetic radiation.

We find that the **intensity** (the number of counts per second) decreases with the square of the distance. This means that if we double the distance, the intensity goes down four times. The relationship is:

$$I = \frac{k(I_0)}{x^2}$$

.... *Equation 18*

[I – intensity (W m^{-2}); I_0 – intensity at the source (W m^{-2}); k – constant; x – the distance from the source (m)]

The unit for intensity is **watt per square metre** (W m^{-2}).

We can explain the inverse square law by reference to this diagram (*Figure 23*).

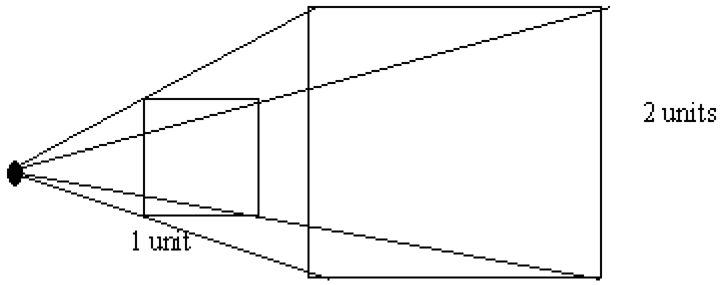


Figure 23 Showing the inverse square law

Let us look at the square, which is one unit away from the source. It has a side of one unit. Suppose we move away from the source by two units. We find that the square has side of length two units. Whereas the area of the little square is 1 square unit, the area of the big square is four square units. So the same amount of energy is spread out over four times as much area.

This is true for any kind of wave radiation, not just gamma.

In three dimensions, we find that waves propagate from a point source **spherically**, but the same argument applies.

Worked Example A sample of radioisotope emits 1.2×10^{12} γ photons per second in all directions. Assuming that the area of a person is about 1 m^2, estimate the number of photons per second received by someone standing (a) 2.0 m; (b) 4.0 m; (c) 5.5 m from the source if they were accidentally exposed.
Answer We need to work out the area of a sphere 2.0 m radius: $A = 4\pi r^2 = 4 \times \pi \times (2.0\text{ m})^2 = 16\text{ m}^2 \times \pi = 50.27\text{ m}^2$ Now we can work out the fraction of the radiation received. Fraction = $1\text{ m}^2 \div 50.27\text{ m}^2 = 0.0199$

Now we can calculate the dose (number of photons per second) received:

$$\text{Dose} = 0.0199 \times 1.2 \times 10^{12}\ \text{s}^{-1} = \underline{\mathbf{2.39 \times 10^{10}\ s^{-1}}}$$

We use the inverse square law to work this out. Double the distance, the intensity goes down by 4 times. So the dose is:

$$\text{Dose} = 2.39 \times 10^{10}\ \text{s}^{-1} \div 4 = \underline{\mathbf{6.00 \times 10^{9}\ s^{-1}}}$$

The problem here is that 5.5 is not an easy multiple of 2. What we need to do is to use:

$$I = \frac{k(I_0)}{x^2}$$

We need to get rid of the constant and the original intensity, so we use the 2.0 m result as a reference:

$$\Rightarrow 2.39 \times 10^{10} = \frac{k(I_0)}{(2.0\ \text{m})^2}$$

And we then write:

$$I = \frac{k(I_0)}{(5.5\text{m})^2}$$

We combine these two to give:

$$2.39 \times 10^{10}\ \text{s}^{-1} \times 4.0\ \text{m}^2 = 30.25\ \text{m}^2 \times I$$

$$\Rightarrow I = \frac{2.39 \times 10^{10}\ \text{s}^{-1} \times 4.00\ \text{m}^2}{30.25\ \text{m}^2} = \underline{\mathbf{3.16 \times 10^{9}\ s^{-1}}}$$

Note that the intensity is, in this case, number of photons per second. If we knew the photon energy, we could work out the intensity in W m^{-2}.

This calculation tells us that the further away from a gamma source, the lower the rate of radiation exposure. This has important implications for work with radioactive materials. The rule is quite simple; the further away, the better. Hence we handle radioactive sources with tongs. School radioactive sources are very weak, but it is still essential to handle them carefully.

12.043 Background Radiation

Radiation detectors will always detect radiation, even if there is no source in the room. There is always **background radiation** coming from all sorts of sources:

- Cosmic rays
- Radioactive material in the bricks of the building.
- Small amounts from medical and industrial uses.
- Yourself (you are alive with Carbon-14 amongst other radioisotopes).

The amount should be constant, but actually it does vary, so when you do experiments, you should always do a background radiation check as a matter of good practice. You should always **subtract** the background radiation from your results to get a **corrected** result.

In some parts of the country, especially where there are granite rocks, the background radiation is quite high. Some houses have fans to reduce the build up of radon gas.

12.044 Intensity

When doing intensity calculations, it is more common that we do not know what I_0 is. Nor do we know the constant k. Neither of these matter. Instead we have a count I_1 at point 1 and a count I_2 at point 2 at distances x_1 and x_2 respectively. So we can write:

$$I_1 = \frac{k(I_0)}{x_1^2}$$

.... *Equation 19*

and

$$I_2 = \frac{k(I_0)}{x_2^2}$$

.... *Equation 20*

We can combine these in a rearranged form to give us:

$$I_1(x_1)^2 = kI_0 = I_2(x_2)^2$$ …………… *Equation 21*

So we can write:

$$I_1x_1^2 = I_2x_2^2$$ .... *Equation 22*

If we rearrange further we can get a ratio:

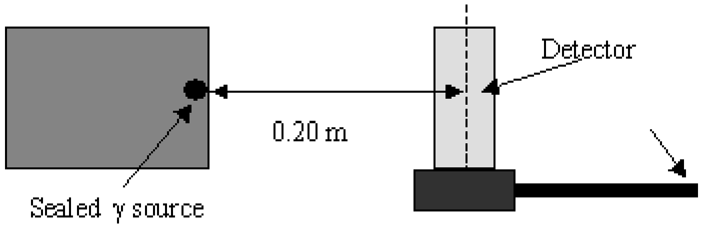
$$\frac{I_1}{I_2} = \left(\frac{x_2}{x_1}\right)^2$$ .... *Equation 23*

This relationship is easier to use and you will have a go at it in question 12.04.1.

Tutorial 12.04 Questions

12.04.1

A detector placed 0.20 m from a sealed gamma ray source receives a mean count rate of 2550 counts per minute. The mean background radiation count is 50 counts per minute. The experiment is set up as shown.



Calculate the least distance between the source and the detector if the count rate is not to exceed 6000 counts per minute.

Tutorial 12.05 Exponential Law of Decay	
All Syllabi	
Contents	
12.051 Radioactive Decay	12.052 Decay Constant
12.053 Exponential Decay	12.054 Nuclear Activity
12.055 Uses of Half-Life	12.056 Radioactive Dating
12.057 Rock Dating	12.058 Calculus Treatment (Extension)

12.051 Radioactive Decay

It is important to understand that radioactive decay is entirely a random and **unpredictable** process. If we look at any one nucleus, it might decay in ten seconds or ten million years. There is no way of telling when the decay will happen, and there is certainly **no way** of speeding up the process. Remember that chemical reactions involve the outer shell electrons; **radioactive decay** involves the **nucleus**.

However when we have many millions of millions of nuclei, we can apply statistical models of **probability**. The **radioactive decay law** states that:

The probability per unit time of a nucleus decaying is a constant, independent of the time.

The **rate** of disintegration of any nuclide at any time is directly proportional to the number of atoms left at the time:

$$\frac{dN}{dt} \propto -N$$

.... *Equation 24*

This is calculus notation for **rate of change in number**.

The minus sign tells us that N decreases as time increases.

12.052 Decay Constant

There is a constant of proportionality called the **radioactive decay constant**, which is given the physics code λ (Greek letter 'l', not to be confused with wavelength), and has the units s^{-1}.

So we can write:

$$\frac{dN}{dt} = (-)\lambda N$$ .... *Equation 25*

The radioactive decay constant is defined as:

the fraction of the total number of nuclei present that decays per unit time, provided that the time interval is small.

It can also be described as the **probability** of the decay of a nucleus per unit time.

For nuclides with relatively long half-lives, the decay constant will have units "per second". However some very unstable nuclides decay in microseconds, so the decay constant would have to be "per nanosecond".

We can so write *Equation 25* in terms of the activity, A. The activity is the number of **decays per second**.

$$A = \lambda N$$ .... *Equation 26*

The unit **Becquerel** (Bq) is often used. 1 Bq = 1 count per second.

Worked Example
0.25 kg radon-226 emits alpha particles at a measured rate of 9.0×10^{12} s^{-1}. What is the decay constant of radium? (No of atoms in a mole = 6.0×10^{23})
Answer Work out the number of particles: 1 mole of radon atoms has a mass of 0.226 kg. $\frac{0.250}{0.226} \times 6.0 \times 10^{23} = 6.64 \times 10^{23}$ atoms We know that the rate of decay is 9.0×10^{12} s^{-1}. So we use $DN/Dt = -\lambda N$ -9×10^{12} s^{-1} $= -\lambda \times 6.64 \times 10^{23}$ $\lambda =$ **1.36×10^{-11} s^{-1}** (The minus sign indicates a decay)

12.053 Exponential Decay

Over longer periods of time, the relationship above does not hold. It can be shown by **calculus** methods (which we will look at later), that the decay follows the relationship:

$$N = N_0 e^{-\lambda t}$$

.... *Equation 27*

[N – no of nuclei; N_0 – original number of nuclei; e – exponential number, 2.718...; λ - decay constant (s^{-1}); t – time (s)]

This relationship is described as an **exponential decay** and the graph looks like this (*Figure 24)*

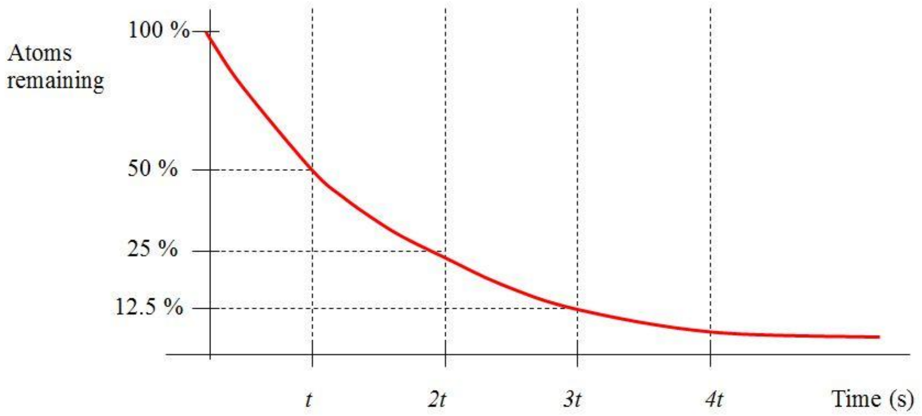


Figure 24 Exponential Decay graph

We should note the following:

- The **rate of decay** represents the **number of atoms remaining**. So we can use this graph for representation of the count rate, or ionisation current.
- The rate of decay is the number of **disintegrations per second**, or the **activity**. It is measured in **Becquerels** (Bq).
- 1 Bq = 1 disintegration per second.

Notice that the horizontal axis is calibrated in periods of time in which the decay goes from an initial value to half that value. This period is called the **half-life**. The formal definition of half-life is the time taken for the activity of a sample to decrease to half some initial value. After 1 half life the activity is 50 %, between 1 and 2 half lives it falls from 50 % to 25 % and so on. If we have a whole number of half-lives, we can do an iterative calculation of the fraction remaining, *i.e.* ½, ¼, 1/8, *etc*.

We can relate the half-life to the decay constant. By definition:

$$\frac{N}{N_0} = 0.5$$

.... *Equation 28*

The formula is:

$$N = N_0 e^{-\lambda t}$$ ………… *Equation 29*

Rearranging gives us

$$\frac{N}{N_0} = e^{-\lambda t}$$

.... *Equation 30*

The term $t_{1/2}$ (pronounced "T-half") is the physics code for **half-life**. So we can write:

$$0.5\frac{N_0}{N_0} = e^{-\lambda t_{1/2}}$$

.... *Equation 31*

Therefore:

$$0.5 = e^{-\lambda t_{1/2}}$$

.... *Equation 32*

Now take **natural** logarithms:

$$\ln 0.5 = -\lambda t_{1/2}$$

.... *Equation 33*

Rearranging:

$$t_{1/2} = \frac{\ln 0.5}{-\lambda} = \frac{-0.693}{-\lambda}$$

.... *Equation 34*

The minus signs cancel out:

$$t_{1/2} = \frac{0.693}{\lambda}$$

.... *Equation 35*

So we can finally write:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

.... *Equation 36*

This will work for any units of time, although we need to be consistent.

Remember that natural logarithms ($\log_e$, or ln) must be used, NOT $\log_{10}$. It is up to you to make sure that you key in the correct data into your calculator.

Maths Note

To work with these relationships, you must be familiar with the concept of **logarithms** (a way of expressing a number as a power of 10 or *e*). You will see logarithms expressed as 'lg' or $\log_{10}$, which means log to the base 10, or 'ln' or $\log_e$, log to the base *e*. The number *e* is the **exponential number**, 2.718...

The latter are known as **natural logarithms**, which are at the heart of exponential functions.

Worked Example

A radiographer has calculated that a patient is to be injected with 1×10^{18} atoms of iodine 131 to monitor thyroid activity. The half-life is 8 days.

Calculate:

(a) the radioactive decay constant

(b) the initial activity

(c) the number of undecayed atoms of iodine 131 after 24 days.

(d) The total activity after 3 days.

Answer

(a) We need to use:

$$t_{1/2} = \frac{0.693}{\lambda}$$

We need to convert the 8 days into seconds.

$$\lambda = \frac{0.693}{8 \text{ dy} \times 86400 \text{ s dy}^{-1}}$$

$$= \mathbf{1.00 \times 10^{-6}\ s^{-1}}$$

(b) Use:

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$DN/Dt = 1.00 \times 10^{-6} \text{ s}^{-1} \times 1 \times 10^{18} = \underline{\mathbf{1 \times 10^{12}\ Bq}}$$

(c) 24 days is 3 half-lives. Therefore the number atoms remaining undecayed is 1/8 of the original.

$$N = 1 \times 10^{18} \div 8 = \underline{\mathbf{1.25 \times 10^{17}}}$$

(d) 3 is not so easy. We use $A = A_0 e^{-\lambda t}$

$$\Rightarrow A = 1 \times 10^{12} \text{ Bq} \times e^{-(1.00 \times 10\text{-}6 \text{ s-}1 \times 3 \times 86400\text{s})}$$

$$\Rightarrow A = 1 \times 10^{12} \text{ Bq} \times e^{-(0.2592)}$$

$$\Rightarrow A = 1 \times 10^{12} \text{ Bq} \times 0.772 = \underline{\mathbf{7.72 \times 10^{11}\ Bq}}$$

[*Alternative method*] If you are not so confident in the use of the natural logarithm, you can work out the number of half-lives a particular time gives. In this case 3 days = 3/8 of a half life.

Activity = $1 \times 10^{12} \times 1/2^{3/8} = 1 \times 10^{12} \times 1/1.30 =$ **7.7×10^{11} Bq**

This is a perfectly valid alternative approach.

12.054 Nuclear Activity

Often we write in terms of the **activity** of a source. This is the number of disintegrations per second, rather than the number of undecayed nuclei. The activity can be written as:

$$A = \frac{dN}{dt}$$

.... *Equation 37*

If we know the activity of the sample, A, and the reference activity, A_0, we can use the equation:

$$A = A_0 e^{-\lambda t}$$

.... *Equation 38*

12.055 Uses of Half-Life

The half-life has important implications for the storage of **radioactive waste**.

Radioactive waste is some of the nastiest muck known to man, so it has to be stored carefully. Isotopes with a short half-life have intense activities, whereas those with a long half-life have lower activities, but it takes much longer for the activity to decay to a reasonable level. Either way, it's not very nice.

Even when there have been several half-lives, there remains quite a considerable activity. If in our example above the activity had decayed to 1/1000th of its original value, we would still have 10^9 disintegrations per second, which is quite a lot.

Another use of the decay equations is in **radioactive dating**. Isotopes used are carbon-14, rubidium-87, and hydrogen 3.

12.056 Radioactive Dating

Carbon-14 has a half-life of 5700 years. It decays by beta minus decay by this equation:

$$^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e^{-} + \overline{\nu_e}$$

.... *Equation 39*

Carbon-14 is made by the collision of an energetic neutron with a nitrogen nucleus, which results in the ejection of a proton. The energetic neutron is formed by cosmic ray activity. Therefore carbon-14 is being made all the time. The rate of decay is balanced by the rate of formation, so the amount of carbon-14 remains almost constant.

The proportion of carbon-14 in living material is very low. For each gram of carbon, the count-rate is about 140 min^{-1}, 2.33 Bq. The count rate is therefore 2300 Bq kg^{-1}. This is the reference point we use for 100 % activity.

If we know the activity of the sample, A, and the reference activity, A_0, we can use the equation:

$$A = A_0 e^{-\lambda t}$$ .... *Equation 40*

We can take natural logs to give us:

$$\ln A = \ln A_0 - \lambda t$$ .... *Equation 41*

And this rearranges to:

$$-\lambda t = \ln A - \ln A_0$$ ……………. *Equation 42*

Since $\ln A_0$ is bigger than $\ln A$, the right hand side of the equation is negative. The left hand side is negative, so the two negative signs cancel. This ensures that the time is positive. We can only go forward in time.

To work out the decay constant, we need to use the equation:

$$\lambda = \frac{0.693}{t_{1/2}}$$

.... *Equation 43*

In this case the half life is 5700 years, which we have to convert to seconds. You do know how to do this, don't you?

Worked example In an archaeological dig, remains are found of a warrior who was buried using a wooden boat for a coffin. A 0.50 gram sample of the boat is found to have an activity of 3800 counts per hour. What is the age of the boat? The count rate for living material is 2300 Bq kg^{-1}. The activity of carbon-14 in living material is 2300 Bq kg^{-1} and the half-life of carbon-14 is 5700 years.
Answer Find out the activity of the sample in Becquerel. $A = 3800\ h^{-1} \div 3600\ s\ h^{-1} = 1.056\ Bq$ Convert this to activity per unit mass. $A = 1.056\ s^{-1} \div 0.5 \times 10^{-3}\ kg = 2111\ Bq\ kg^{-1}$ Now we need to work out the decay constant. $\lambda = 0.693 \div (5700\ y \times 365\ dy \times 24\ h \times 3600\ s) = 3.86 \times 10^{-12}\ s^{-1}$ Now use: $-\lambda t = \ln A - \ln A_0$ $-3.86 \times 10^{-12}\ s^{-1} \times t = \ln (2111\ Bq\ kg^{-1}) - \ln (2300\ Bq\ kg^{-1})$ $-3.86 \times 10^{-12}\ s^{-1} \times t = 7.655 - 7.741 = -0.0857$ Notice that the minus signs cancel: $t = 0.0857 \div 3.86 \times 10^{-12} =$ **2.2×10^{10} s** (700 years) Normally you would leave your answer in seconds. The answer is given to 2 significant figures as the "activity in living material" data item is to 2 significant figures.

The calculation above assumes that the level of carbon-14 is the same in the year 1300 as it is now. If the cosmic ray activity were less then, the amount of carbon 14 would be reduced. The warrior's remains would be more recent.

12.057 Rock Dating

Carbon-14 dating is only useful for objects under about 50000 years old. It's no good for dating rocks. The isotope potassium-40 is useful. There are two modes of decay for potassium-40:

1. Normal beta minus decay;
2. Electron Capture.

For beta minus, the decay is:

$$^{40}_{19}K \rightarrow ^{40}_{20}Ca + ^{0}_{-1}e^{-} + \overline{\nu_e}$$

.... *Equation 44*

The half life for this is about 1250 million years.

For electron capture, the electron is captured from the inner shell:

$$^{40}_{19}K + ^{0}_{-1}e^{-} \rightarrow ^{40}_{18}Ar + \nu_e$$

.... *Equation 45*

About 1 in 8 decays is of the latter type. So out of 9 potassium-40 atoms, 8 will decay to calcium, while 1 decays to argon. The problem with the decay to calcium is that calcium is as common as muck, so we wouldn't be able to tell which calcium atom came from the potassium decay, and which came from other sources.

However, Argon is an inert gas, and does not react with any crystals. Argon atoms remain trapped in the rock, so if we knew how many argon atoms there were, we could work out how many potassium atoms had decayed. So if we counted 12×10^6 argon atoms, we could say that these resulted from 108 × 10^6 potassium 40 atoms. So if we have 500 × 10^6 potassium 40 atoms left, we can say that we would have started off with 108 × 10^6 + 500 × 10^6 = 608 × 10^6 atoms.

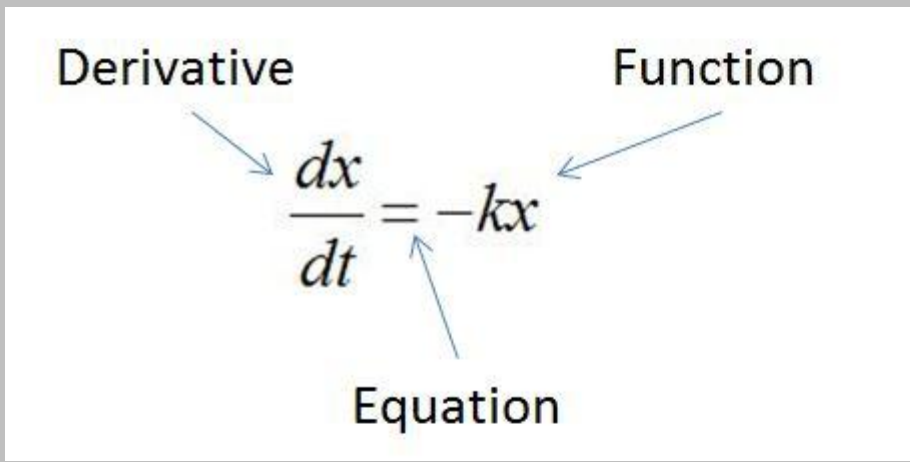
12.058 Calculus Treatment of Radioactive Decay (Extension only)

You are NOT expected to use calculus at this level, so do NOT attempt to show that you are clever by using it unless you really know what you are doing! However, if you go on to study Physics at University level, you will need to know about calculus.

You may have seen the calculus derivation of the exponential equation for discharge of a capacitor. Exponential decay of unstable nuclei is similar.

Maths note

A **differential equation** is where there is a function combined with a derivative, for example:



Differential equations describe the way that values change when a system changes by a constant fraction. The solution is not always easy, but a mathematical technique called the **separation of variables** is useful. To do this we:

1. Multiply both sides by dt;
2. Divide both sides by x.

This gives us:

$$\frac{dx}{x} = k\,dt$$

Then we need to integrate both sides to get rid of the derivative.

$$\int \frac{dx}{x} = \int k \, dt$$

The problem here is that the power rule doesn't work:

$$\int x^{-1} \, dx = \frac{x^0}{0} + C$$

which is impossible. Instead there is another rule:

$$\int x^{-1} \, dx = \ln x + C$$

So we do the first integration:

$$\ln(x) + C = \int k \, dt$$

And the second (using the power rule since $k = kx^0$):

$$\ln(x) + C = kt + D$$

C and D are constants of two different values. We can, for the sake of this argument, roll the two constants into one by writing:

$$a = D - C$$

So we rewrite the equation as:

$$\ln(x) = kt + a$$

Now we bring in the exponential number e (= 2.718...) on both sides to get rid of the natural logarithm (**ln** is a logarithm to the base e).

The terms become **exponents**. (An exponent is a power to which a base number is raised, i.e. e^{exponent}.) We rewrite the equation as:

$$e^{(\ln(x))} = e^{(kt + a)} = e^{kt} \times e^{a}$$

The term e^a is simply a constant. Since $e^{(\ln(x))} = x$, so we now write the equation as:

$$x = e^{kt} \times e^{a}$$

Note that when two numbers add up, they **multiply** when they become exponents. This is the converse of when logarithms of two multiplied numbers add up.

If the constants C and D are zero, $\mathrm{e}^{\mathrm{a}} = \mathrm{e}^{0} = 1$. Therefore the expression becomes:

$$x = e^{kt}$$

We know that:

$$A = \lambda N$$ .... *Equation 46*

We can rewrite this in calculus notation:

$$\frac{dN}{dt} = -\lambda N$$

.... *Equation 47*

We can apply the method of separation of variables to the differential equation above. In this case the term λ is the fractional amount by which the charge decays. So to solve this differential equation, we multiply by dt and divide by N.

So our equation becomes:

$$\frac{dN}{N} = -\lambda\, dt$$

.... *Equation 48*

Now we integrate both sides:

$$\int \frac{1}{N} dN = \int -\lambda\, dt$$

.... *Equation 49*

On the right hand side, there doesn't appear to be anything to integrate with reference to t. The term λ is a constant. However we can rewrite the equation as:

$$\int \frac{1}{N} dN = \int -\lambda \times 1\, dt$$

.... *Equation 50*

The trick here is that $1 = t^0$ so we can rewrite the equation further:

$$\int \frac{1}{N} dN = \int -\lambda t^0\, dt$$

.... *Equation 51*

So we can carry out the integration, applying the rules in the Maths Note above:

$$\ln(N) + C = -\lambda t + D$$

.... *Equation 52*

We can combine the constants by saying:

$$a = D - C$$ …………… *Equation 53*

So we now write:

$$\ln(N) = -\lambda t + a$$

.... *Equation 54*

To get rid of the natural logarithm, we make both sides an exponent of the exponential number e (=2.718...). Now $e^{\ln(N)} = N$, so our equation now becomes:

$$N = e^{-\lambda t} \times e^{a}$$

.... *Equation 55*

Now when $t = 0$, we can see that:

$$N = e^{-\lambda \times 0} \times e^{a} = 1 \times e^{a}$$

.... *Equation 56*

So far we have haven't attempted to define what e^{a} is. But we can easily see that it is **number at time 0**. This has the physics code N_0, so we can rewrite the equation as:

$$N = N_0 e^{-\lambda t}$$

.... *Equation 57*

This is our final result.

Tutorial 12.05 Questions

12.05.1

A sample of living material contains carbon 14 with an activity of 260 Bq kg^{-1}. What is the decay constant?

(The fraction that is made of carbon-14 is 1.4×10^{-12})

12.05.2

What is the half life of radon-226? ($\lambda = 1.36 \times 10^{-11}\ s^{-1}$)

12.05.3

Strontium-90 is a beta emitter. It is one of the radio-nuclides found in the fall out from an atomic bomb explosion. It can be absorbed into the bone. It emits beta particles and has a half life of 28 years. What is the time needed for the activity to fall to 5 % of the original?

12.05.4

A GM tube placed close to a radium source gives an initial average corrected count rate of 334 s^{-1}

(a) The GM tube detects 10 % of the radiation. What is the initial activity?

(b) Initially there were 1.5×10^9 nuclei in the sample. What is the decay constant?

(c) What is the half life of the radium in days?

12.05.5

A sample of rock has 12×10^6 argon atoms. Scientists have calculated that these resulted from 108×10^6 potassium 40 atoms. They work out that from 500×10^6 potassium 40 atoms left, they would have started off with $108 \times 10^6 + 500 \times 10^6 = 608 \times 10^6$ atoms.

If potassium-40 has a half-life of 1250 million years, what is the age of the sample of rock discussed in the paragraph above?

Tutorial 12.06 Nuclear Radius	
All Syllabi	
Contents	
12.061 Electron Diffraction	12.062 X-ray Diffraction
12.063 Nuclear radius	12.064 Nuclear Density

12.061 Electron Diffraction

The key idea to the use of particles and radiation to investigate the structure of matter lies in the **de Broglie** (pronounced 'de Broy') relationship, which states that particles have wave properties. It is the logical extension of the particulate nature of electromagnetic wave phenomena.

When we investigate large objects, light is a good way to do this. We have eyes to see with, and we can make direct observations with a magnifying lens or a microscope. On the atomic and nuclear level, microscopic objects like fleas and bacteria are very large objects indeed. However the light is limited by its wavelength to resolving objects about 1 μm across. Much less than that, then diffraction becomes important. Waves will not travel through a gap less than a wavelength.

Electrons can be shown to have wave properties by the simple use of an **electron diffraction tube**. A slice of carbon is placed in a beam of electrons so that the electrons diffract (*Figure 25*).

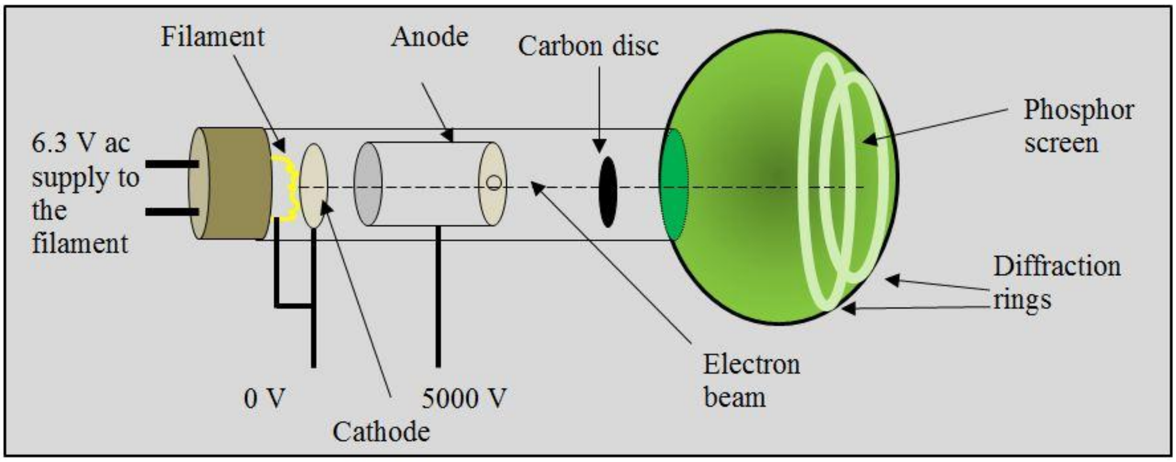


Figure 25 Electron diffraction

This has led to the development of the **electron microscope**, which allows magnifications much bigger than was ever possible with the light microscope. A good light microscope can magnify up to 1000 times. The electron microscope can magnify up to about 1 million times, and can reveal the existence of individual **atoms**. The electron beams are focused by magnets just like the lenses on a microscope.

The existence of the nucleus was determined about 130 years ago by Ernest Rutherford (1871 – 1937) in his famous alpha scattering experiment (*Figure 26*).

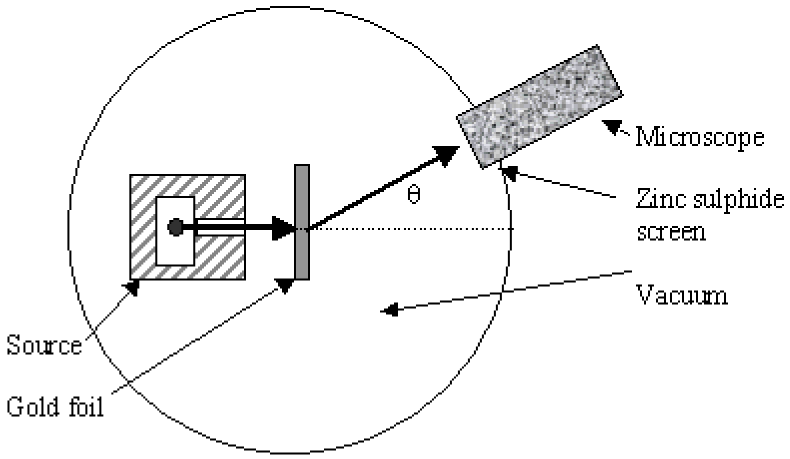


Figure 26 Rutherford's alpha scattering experiment

The idea that inspired the experiment was that the alpha particles were considered to be nuclear sized bullets that would smash the atoms in the gold foil like water melons. Instead they found that many alpha particles were deflected, while a few came back in the direction they came from. It led to two important conclusions:

- There is a positively charged nucleus
- The nucleus is very tiny compared with the rest of the atom; most of the atom is just empty space.

Further calculations led to an estimate of nuclear size of about 1×10^{-15} m or **1 femtometre** (fm).

12.062 X-ray Diffraction

X ray diffraction has been a useful tool to discover the structure of solid materials. It was perfected by the father and son team of William and Lawrence Bragg. X rays are of course electromagnetic waves, which are scattered by diffraction by the crystal lattices of materials. A sample of the material is placed in the beam of X-rays, and the resulting scattering pattern is picked up on a photographic plate. The X rays are diffracted in a cone (*Figure 27*).

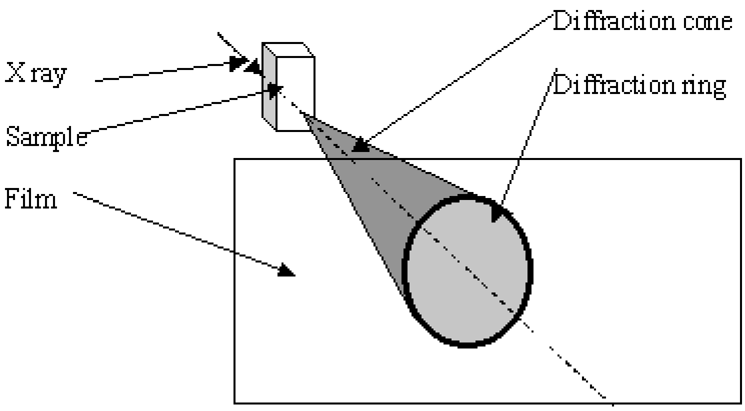


Figure 27 X-ray diffraction

By use of a simple equation we can determine the separation of layers of atoms. The equation (which you won't be asked about in the AQA exam) is:

$$n\lambda = 2d \sin \theta$$ .... *Equation 58*

There is a simple student experiment in which microwaves are used in conjunction with a lattice of 4 cm polystyrene spheres. It gives students experience in the use of this equation to determine the layer separation.

More complex analysis is needed for determination of the structure of the crystal lattices, and this is covered in the discipline of **X-ray crystallography**. Some very complicated structures indeed have been worked out using such techniques.

To get the resolutions required to look at the **nucleus**, we need de Broglie wavelengths of 10^{-15} m, and this is not possible with electrons. If we use more massive particles, we can obtain much shorter de Broglie wavelengths, hence more resolution. That's the theory. In practice, the nuclei bombarded with **high energy** particles have tended to break up. It has been described as finding out how a watch works by smashing it with another watch, and guessing how the pieces fit together.

Particle physics experiments have been able to interpret the sub nuclear particles that make up the nucleons within the nucleus. They cannot be seen directly of course, but inference has been drawn from the patterns observed in collisions between particles. Physicists use powerful computers to explain these phenomena.

12.063 Nuclear radius

Rutherford estimated the radius of a nucleus as 3.0×10^{-14} m from the data obtained in alpha scattering experiments. He used Coulomb's law in his calculations (*Figure 28*).

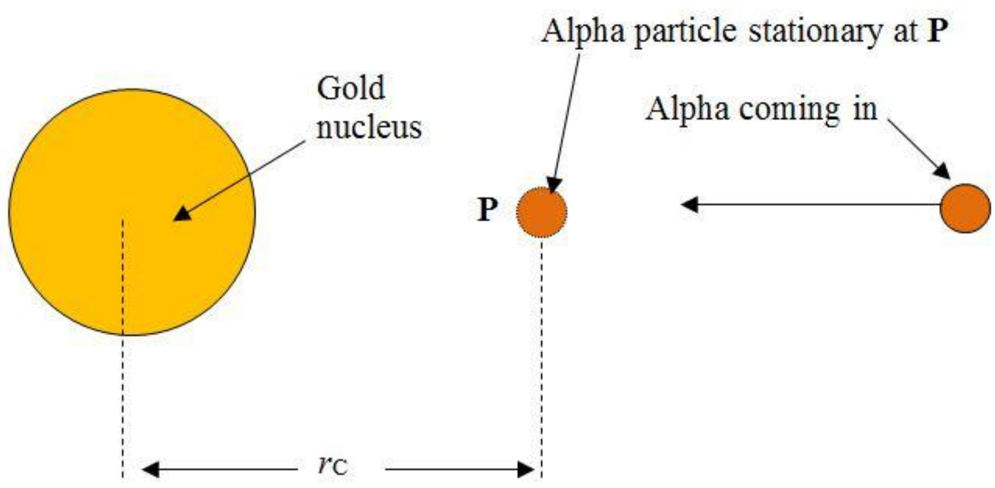


Figure 28 Rutherford's estimation of the size of a gold nucleus

The point **P** is the closest that the alpha particle gets to the nucleus before being repelled. Its kinetic energy is 0 because it is stationary. All its energy is potential. So we can use our knowledge of electrostatic potential energy to calculate the distance.

E_p = potential at **P** × charge of the alpha particle

$$E_{\mathrm{p}} = \frac{1}{4\pi\varepsilon_0}\frac{Ze \times 2e}{r_{\mathrm{c}}}$$

.... *Equation 59*

Rearranging gives us:

$$r_{\mathrm{c}} = \frac{1}{4\pi\varepsilon_0}\frac{Ze \times 2e}{E_{\mathrm{p}}}$$

.... *Equation 60*

This calculation is based on alpha particles of 7.68 MeV energy which converts to 1.23 × 10^{-12} J. Therefore:

$$r_{\mathrm{c}} = 9.0 \times 10^{9}\mathrm{m\ F^{-1}} \times \frac{2 \times 1.6 \times 10^{-19}\mathrm{C} \times 79 \times 10^{-19}\mathrm{C}}{1.23 \times 10^{-12}\mathrm{J}}$$

.... *Equation 61*

r_{c} = 2.96 × 10^{-14} m = **3.0 × 10^{-14} m** (2 s.f.)

There are one or two points to bear in mind from this calculation:

- The nucleus is treated as a **point charge**. At this level it is not.
- The alpha particles are stopped some distance away from the nucleus.
- It takes higher energy alpha particles to penetrate the nucleus.
- The values for the nuclear radius given by other particles such as protons, neutrons and electrons are slightly different.

A more accurate estimate of the nuclear radius has been determined by the use of a technique called **electron scattering**. The electrons interact with the nucleus entirely by the electromagnetic interaction whereas the alpha particles interact by the strong nuclear interaction, which is not well understood.

The scattering of electrons is treated like the diffraction of waves around a spherical object. The in depth analysis of the results is quite complicated, but a reasonable estimate can be obtained with a relatively simple equation (which you don't need for the exam):

$$\sin\theta = \frac{0.61\lambda}{R}$$

.... *Equation 62*

The term λ is the **de Broglie wavelength** of the high-energy electrons, θ is the angle of diffraction, and R is the nuclear radius.

This gives a result of the nuclear radius being **2.65 × 10^{-15} m**.

We need to note the following:

- To get an appreciable electron scattering effect, we need to have electrons with a de Broglie wavelength of about the nuclear diameter. This requires very high energies.
- The electron diffraction minima are not zero, indicating that the boundary of the nucleus is fuzzy, not sharp.
- Since the boundary is not sharp, various methods of determining the radius give rather variable results, from 1.2 fm to 1.5 fm.

The radius depends on the nucleon number through a simple relationship:

$$R = r_0 A^{\frac{1}{3}}$$

.... *Equation 63*

[The term $A^{1/3}$ means the cube root of A, the nucleon number. The term r_0 is a constant with the value **1.4 × 10^{-15} m**. R is the nuclear radius.]

Worked Example What is the nuclear radius of gold with a nucleon number of 197?
Answer $R = r_0 A^{1/3}$ = 1.4 × 10^{-15} m × $\sqrt[3]{197}$ = 1.4 × 10^{-15} m × 5.82 = **8.15 × 10^{-15} m**

We can plot the radius against the cube root of the nucleon number, which gives us a straight line plot as shown on the graph (*Figure 29*).

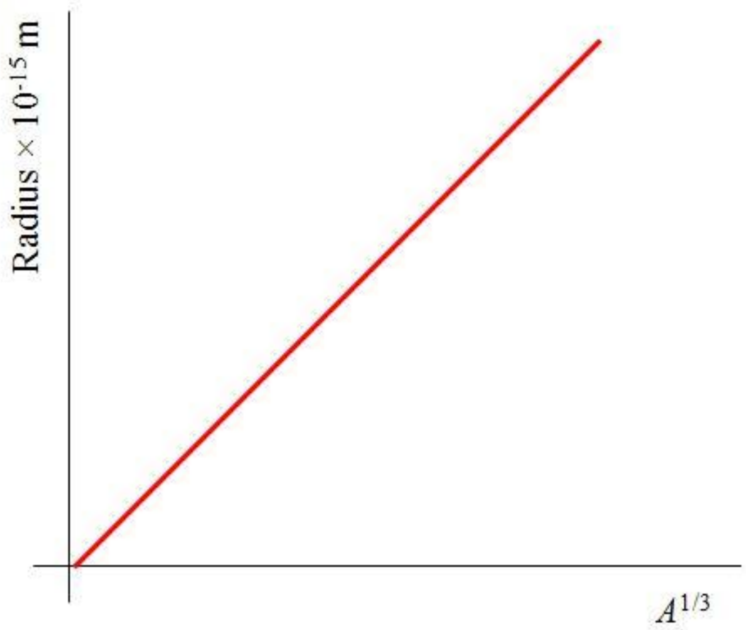


Figure 29 Plot of radius against cube root of the nucleon number

So we can see a nice linear progression. We need to note one or two points.

- The constant r_0 is variable depending on the type of particle used.
- The relationship $R = r_0 A^{1/3}$ is always true, whatever the method used.
- Be careful not to confuse the **nuclear** radius with the **atomic** radius. The atomic radius is remarkably similar, whether the element is light or heavy. This would make sense as the nucleus occupies such a small fraction of the space in an atom.
- Remember to use the **nucleon** number, not the proton number.

If we plot $\ln R$ against $\ln A$, we get a straight line. The intercept is r_0 (*Figure 30*)

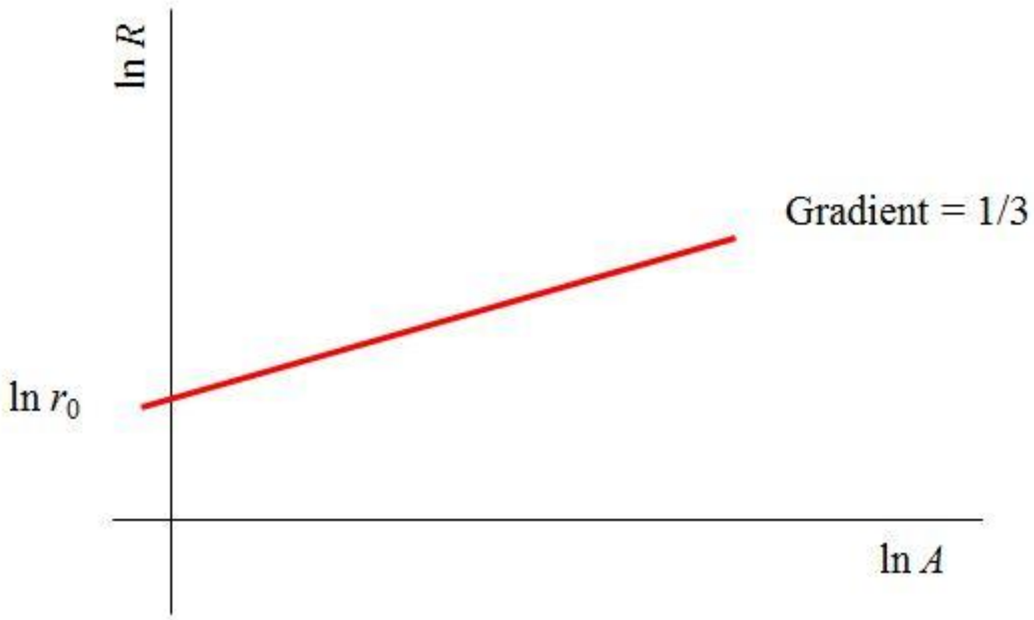


Figure 30 Plotting ln R against ln A

12.064 Nuclear Density

We are used to the **density** (mass per unit volume) of solid materials being in the order of 1000 to 10000 kg m^{-3}. Since most of the atom is empty space, and the vast majority of the matter in an atom is contained in the tiny space of a nucleus, it makes sense that the density of nuclear material is very high indeed.

The calculation in 12.06.5 gives you a really big number, which works out at about 100 million tonnes per cubic centimetre. Drop that on your foot, and you really will have tears in your eyes.

Put another way, the mass of the Earth would fit in a sphere of radius 100 metres; it would fit on top of your school.

When large stars die, they can do so in a cataclysmic explosion called a **supernova**. Strange things happen: the remains of the star collapse in on themselves under gravity, and the material is squashed so much that electrons are absorbed by protons to make neutrons. The process of **electron degeneracy** stops further collapse. **Neutron stars** are the result, which have an intense gravity field. If the star is bigger, then the neutrons are squashed by the force of gravity into a really tiny space until stopped by **neutron degeneracy**. Really big stars collapse in such a way that even neutron degeneracy will stop it. Here the laws of physics no longer apply, and even light cannot escape. We have a **black hole**.

Tutorial 12.06 Questions

12.06.1

Use Coulomb's Law to work out the force of repulsion between two protons 1×10^{-15} m apart. Comment on your result.

12.06.2

Use the value you worked out in Question 1 to comment on the magnitude of the strong force. Explain your answer.

12.06.3

Describe the principal features of the nuclear model of the atom suggested by Rutherford.

12.06.4

What is the nuclear radius of an iron atom, of which the nucleus contains 56 nucleons? Would it be any different to a cobalt-56 nucleus?

12.06.5

What is the nuclear density of an iron atom, of which the nucleus contains 56 nucleons?

Mass of a nucleon = 1.67×10^{-27} kg

2. Nuclear Energy

Tutorial 12.07 Mass and Energy

All Syllabi

Contents

12.071 The Atomic Mass Unit

Kilograms are useful for measuring large masses, but like many of the SI units, on the atomic scale they are far too big. It's a bit like a model maker using kilometres rather than millimetres.

The **atomic mass unit** is far more convenient to use with nuclear masses. It uses carbon 12 as a reference and is defined as:

Exactly 1/12th the mass of a carbon 12 atom

The **relative atomic mass** of an atom is useful. It is defined as:

$$\frac{\textbf{Mass of the atom}}{\textbf{Mass of carbon 12 atom}} \times 12$$

The relative atomic mass of atoms is usually very close to a whole number, which is consistent with the idea of a nucleus made up of whole numbers of nucleons.

1 atomic mass unit (u) = **1.661×10^{-27} kg**.

Do not confuse the atomic mass units with the mass of a nucleon:
Mass of nucleon = 1.67×10^{-27} kg

The table shows particle masses in atomic mass units. Note that the numbers are expressed to a large number of significant figures as the changes are quite subtle.

Particle	***Mass (u)***
Electron	0.000549
Neutron	1.008665
Proton	1.007276
Hydrogen atom ($1p^+ + 1\ e^-$)	1.007825
Helium atom ($2p^+ + 2\ n + 2e^-$)	4.002063
a particle ($2p^+ + 2\ n$)	4.001505

We need to be careful to distinguish between the **atomic mass** and the **nuclear mass**.

- The atomic mass is the mass of an atom complete with its electrons;
- The nuclear mass is the mass of the nucleus alone. To get the nuclear mass we need to take away the mass of the electrons.

12.072 Mass Defect

If we add together the mass of an electron and the mass of a single proton, we get the mass of a hydrogen atom. Let us do the same for a helium atom.

Particle	Mass (u)	Number	Total (u)
Proton	1.007276	2	2.014552
Neutron	1.008665	2	2.017330
Electron	0.000549	2	0.001098
			4.032980

However, if we look in a data book, we see that the atomic mass is 4.002603 u. There is a difference of 0.030377 u.

All atoms are lighter than the sum of the masses of the protons, electrons, and neutrons. This is the **mass defect**, which is the difference between the total mass of the nucleons and the measured mass of the nucleus itself.

To extract a proton or a neutron from the nucleus, we have to pull pretty hard. Then we find that it will regain its missing mass. We can use the idea of **binding energy** to explain this. The binding energy is defined as the energy released when a nucleus is assembled from its constituent nucleons. It is equal to the energy needed to tear the nucleus apart into its nucleons.

So with our helium atom, the missing 0.030377 u is released when the nucleons come together. That energy has to be put back to split the nucleus up again.

This brings us onto the strange idea that mass and energy at the nuclear level are interchangeable and can be related with Einstein's simple equation:

$$E = mc^2$$ .... *Equation 64*

[E – energy (J); m – mass (kg); c – speed of light (m s^{-1})]

It is important to convert the mass from atomic mass units to kilograms. The answer we get is in joules. We can convert this to eV by dividing by 1.6×10^{-19} J eV^{-1}.

You may see the equation written as:

$$E = \Delta mc^2$$

.... *Equation 65*

The term Δm represents the change in mass.

Joules are not convenient units to use at the nuclear level, so we convert to electron volts (eV) by dividing by 1.6×10^{-19} J eV^{-1}.

A useful conversion factor between mass and energy is that 1 u = 931.3 MeV.

Worked Example

Particle	**Mass (u)**	**Number**	**Total (u)**
Proton	1.007276	3	3.021828
Neutron	1.008665	4	4.03466

What is the mass defect in atomic mass units (u) and in kilograms for the lithium nucleus which has 7 nucleons, and a proton number of 3? What is the binding energy in J and eV? What is the binding energy per nucleon in eV? The nuclear mass = 7.014353 u.

Answer

Li has a nucleon number of 7 and a proton number of 3, which means there are 3 protons and 4 neutrons.

Now look up the masses for the proton and neutron from the data. These will be given to you; you are not expected to remember them.

Add them together to get 7.056488 u

Now take away the nuclear mass from the number above to get the mass deficit.

7.056488 u - 7.014353 u = 0.042135 u

Now convert to kilograms: 1 u = 1.661×10^{-27} kg

0.042135 u × 1.661×10^{-27} kg = $6.9986235 \times 10^{-29}$ kg

Now use $E = mc^2$ to work out the binding energy:

E = 6.9986235 × 10^{-29} kg × (3.0 × 10^8 m s^{-1})^2 = **6.3 × 10^{-12} J**

In electron volts, this is 6.3 × 10^{-12} J ÷ 1.6 × 10^{-19} eV J^{-1} = **3.9 × 10^7 eV** = 39 MeV.

There are 7 nucleons.

binding energy per nucleon = 3.9 × 10^7 eV ÷ 7 = **5.6 × 10^6** eV

12.073 Binding Energy Per Nucleon

If we know the binding energy in a nucleus, and the number of nucleons, we can work out the **binding energy per nucleon**, which is the average energy needed to remove each nucleon. The higher the binding energy per nucleon, the more stable is the nucleus. For helium (^4He) the binding energy per nucleon is:

Binding energy per nucleon = 28.38 MeV ÷ 4 = 7.1 MeV

We can plot a graph of binding energy per nucleon against nucleon number (*Figure 31*).

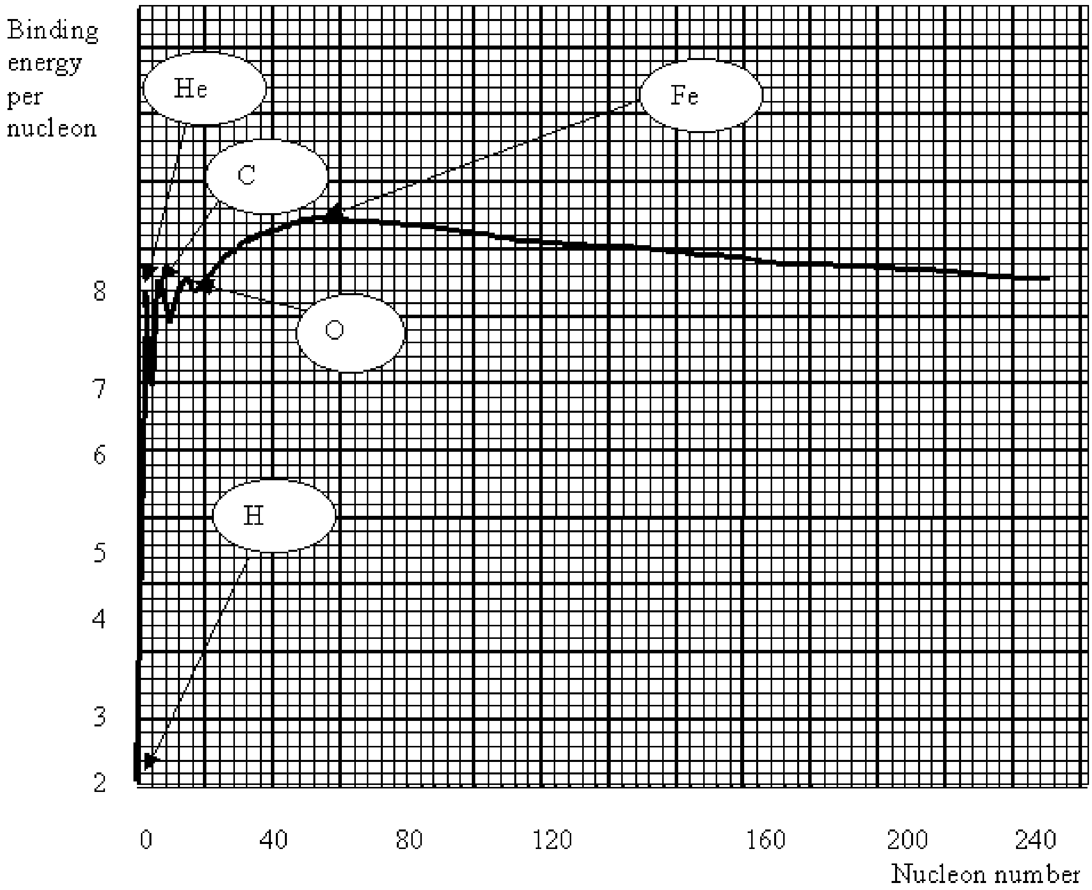


Figure 31 Graph of binding energy per nucleon against nucleon number

From this graph we can see the following:

- The vast majority of nuclides have a binding energy of 8 MeV per nucleon.
- Helium has a particularly high value of binding energy per nucleon, much higher than the light isotopes of hydrogen.
- There is a trend for nuclides of nucleon numbers in multiples of 4 to be particularly stable (i.e. have a high binding energy).
- Fe is the most stable nuclide.
- The largest nuclides tend to be less stable, with slightly lower binding energies per nucleon.

Iron has the highest binding energy per nucleon so is the most stable nucleus. If we look at large nuclei (greater than iron), we find that the further to the right (greater nucleon number) the less stable the nuclei. This is because the binding energy per nucleon is getting less. The explanation for this observation lies in that the strong nuclear force that binds the nucleus together has a very limited range, and there is a limit to the number of nucleons that can be crammed into a particular space.

12.074 Radioactive Decay and Binding Energy

Radioactive decay happens when an unstable nucleus emits radiation. It becomes more stable. The daughter nuclei always have a higher binding energy per nucleon than the parent nucleus.

Let us look at alpha decay:

$$^{228}_{90}Th \rightarrow {}^{224}_{88}Ra + {}^{4}_{2}\alpha$$ .... *Equation 66*

Mass of the thorium nucleus = 227.97929 u

Mass of the radium nucleus = 223.97189 u

Mass of the alpha particle (helium nucleus) = 4.00151 u

Mass on the left hand side = 227.97929 u

Mass on the right hand side = 223.97189 u + 4.00151 u = 227. 97340 u

The right hand side has a mass defect = 227.97929 u - 227. 97340 u = 0.00589 u

The mass defect can be written in kilograms and the energy can be expressed in joules, but nuclear physicists use a useful little dodge. The energy equivalence of 1 u = 931.5 MeV. So the energy given out by this decay is:

E = 931.5 × 0.00589 = **5.49 MeV**.

12.075 Fission

If we look at the graph with binding energy per nucleon, we observe that the large nuclei have a lower binding energy per nucleon. This means that they are less stable. This lack of stability is usually shown by radioactive decay, which occurs in a predictable way. Very rarely a large nucleus will split up spontaneously into fragments. This splitting of the nucleus is called **fission**.

The easiest way to explain this is to consider the nucleus as a "wobbly drop". Nuclei are not tidy and neatly arranged rows of neutrons and protons; they are microscopic bedlam.

The strong nuclear force acts between neighbouring nucleons (*Figure 32*).

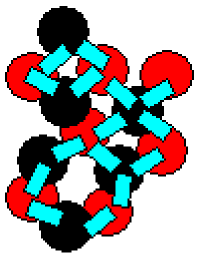


Figure 32 The blue rectangles represent the strong interaction between nucleons

The nucleons are not linked with the same neighbours all the time. Instead they are constantly swapping about. However the enough of the nucleons linked to stop the repulsive electromagnetic force tearing the nucleus apart.

Now we imagine the nucleus as a wobbly drop (*Figure 33*).

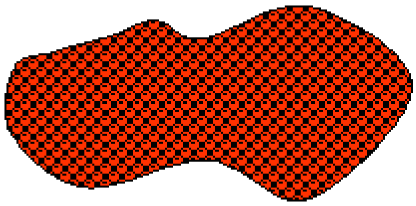


Figure 33 The nucleus as a wobbly drop.

Now if the nucleus gets to this shape (*Figure 34*).

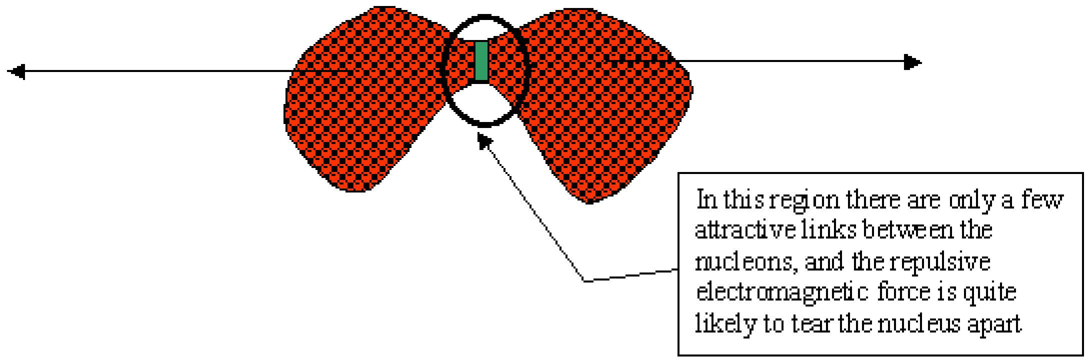


Figure 34 The fissile nucleus about to undergo fission

The nucleus flies apart in two fragments (*Figure 35*).

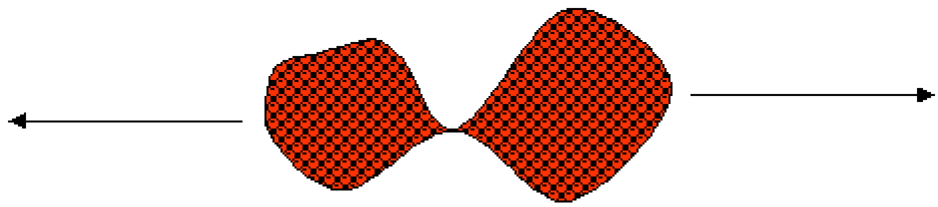


Figure 35 The fissile nucleus flies apart

The detail of the mechanism that drives this process is complicated and is based on Heisenberg's uncertainty principle. A similar model can be used to explain how alpha decay works.

We can induce fission in large nuclei such as uranium-235. The most common isotope of uranium, U-238, does not split easily, but the 235 isotope does. We induce fission by "tickling" the nucleus with a "**thermal**" neutron. The neutron has to have the right kinetic energy:

- Too little kinetic energy means that the neutron will bounce off the nucleus;
- Too much kinetic energy means that the neutron will go right through the nucleus.
- Just right means that the neutron will be **captured** by the strong force, which is attractive between nucleons. The neutron gives the nucleus enough energy to resonate, and this will make the nucleus neck as shown above

The tickled nucleus flies apart into a number of fragments, leaving on average three neutrons left over. These too are able to tickle other nuclei and make them split. Each neutron spawns three more neutrons in each fission, so we get a **chain reaction** (*Figure 36*).

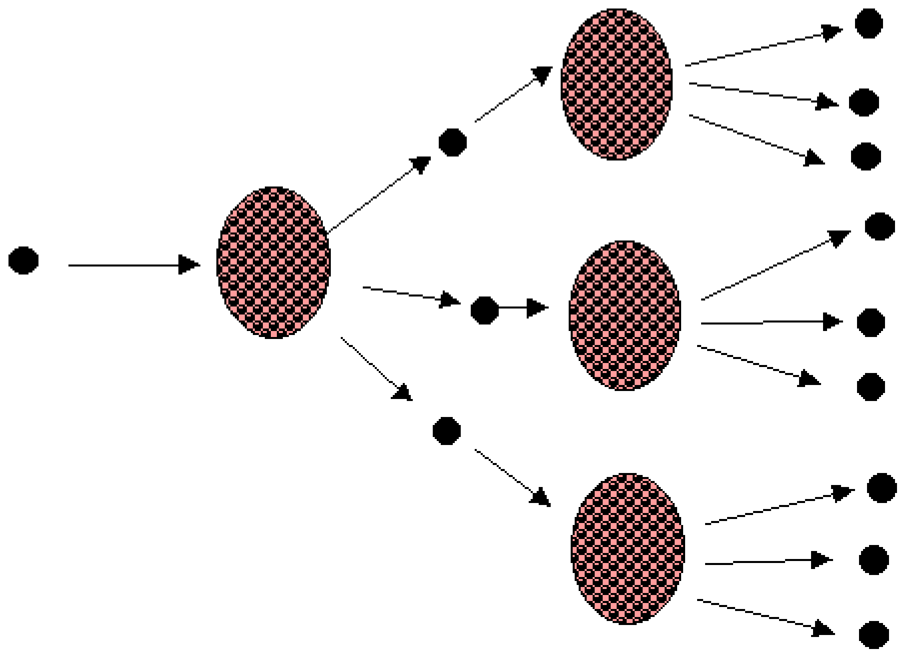


Figure 36 A fission chain reaction

There is a **mass defect** in the products of the fission so energy is given out. In an uncontrolled chain reaction, the energy is given out in the form of a violent explosion, which is many times more powerful than the explosive decomposition of TNT. In an atomic bomb, the mass that is converted to energy is about 20 grams.

Nuclear fission has NOTHING whatever to do with radioactive decay. However the parent nucleus may decay by normal radioactive decay processes, and the daughter nuclei may well be radioactive. This is a common bear trap.

The daughter fragments may well be highly unstable, and decay by radioactivity. These form the dangerous **fall-out** of an atomic bomb detonation, or the **waste** from a nuclear power station. Either way, they form some of the nastiest muck known to mankind.

Compared to the speed of many particles in nuclear and particle physics, the speed in Question 12.07.5 is pretty sluggish.

A common bear trap is to say that nuclei are smashed to pieces by neutrons. The neutrons tickle the nucleus; they do not hammer it.

Some students confuse fission and fusion and use the word "fussion". It will be marked wrong in the exam, so don't.

12.076 Fusion

Fusion means joining nuclei together, every alchemist's dream. It is easier said than done. The idea is that **light nuclei are joined together**, increasing the binding energy per nucleon. This will result in lots of energy being given out. A possible reaction is:

$$^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + \text{energy}$$

..............*Equation 67*

It is not simply a case of sticking some deuterium and tritium together and shaking it up. Each nucleus has to have sufficient energy to:

- Overcome electrostatic repulsion from the protons;
- Overcome the repulsive strong force which is found outside the region of the strong force.

This means that the gases have to be heated to a very high temperature, 100 million Kelvin. As all matter at this temperature exists as an ionised gas (**plasma**), it has to be confined in a very small space by powerful magnetic fields. A considerable amount of effort has been made to make fusion work to generate electricity. A fusion reactor would be made to boil water to turn a turbine. Fusion has occurred, but the energy put in to cook the gases enough to make them fuse is far greater than the energy got out by a fusion reaction.

The only use that fusion has been put to is in a **thermonuclear device**. The third bomb from the left in *Figure 37* is a genuine thermonuclear device, now on display (with the nasty bits taken out). The amount of hydrogen required in the bomb below (430 kilo-tonnes) would fill a small party balloon.

Figure 37 The third bomb from the left is a thermonuclear device

Some scientists claim to have found fusion at low temperatures. They had a strange <u>chemical</u> reaction, but it was not fusion.

Fusion, if it could be made to work, has a number of advantages over fission:

- Greater power per kilogram of fuel used;
- Raw materials are cheap and readily available
- No radioactive elements are made by the reaction.

The downside is that materials that make up the reactor will be irradiated with neutrons which will make them radioactive.

12.077 Fusion in Stars

The process has three stages:

1. Proton + Proton → Deuterium + positron + electron neutrino

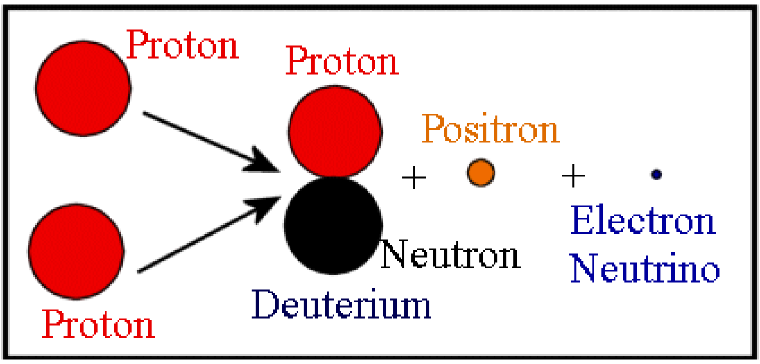


Figure 38 Stage 1 in fusion in stars

2. Deuterium + proton ® Helium 3 + photon

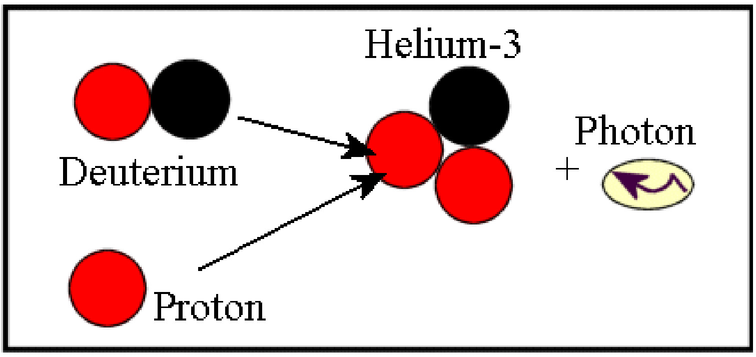


Figure 39 Stage 2 in fusion in stars

3. Helium 3 + Helium 3 ® Helium 4 + proton + proton

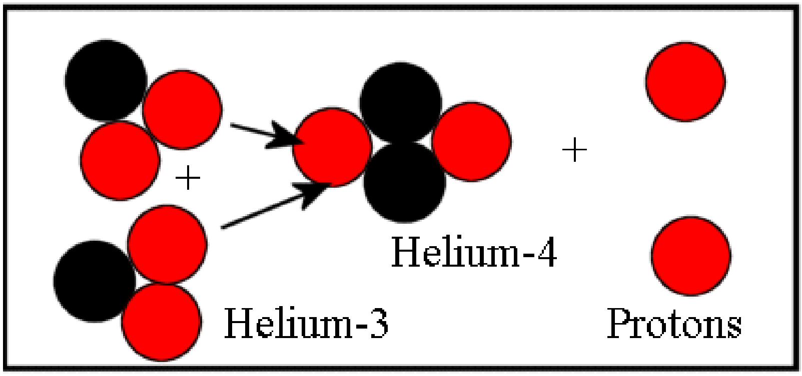


Figure 40 Stage 3 in fusion in stars

Since two protons are left over, the reaction is self sustaining.

Tutorial 12.07 Questions

12.07.1

What is the nuclear mass of helium 3 (^{3}He) of which the atomic mass is 3.016030 u?

12.07.2

What is the binding energy of the helium atom whose mass defect is 0.030377 u?

12.07.3

What is the mass defect in atomic mass units (u) and in kilograms for the copper nucleus which has 63 nucleons, and a proton number of 29? What is the binding energy in J and eV? What is the binding energy per nucleon in eV?

The nuclear mass = 62.91367 u.

12.07.4

This is a beta decay:

$$^{29}_{13}\text{Al} \rightarrow {}^{29}_{14}\text{Si} + {}^{0}_{-1}e^{-} + {}^{0}_{0}\overline{\nu}_e$$

Given these data:

Mass of aluminium nucleus = 28.97330 u

Mass of silicon nucleus = 28.96880 u

Mass of beta particle (electron) = 0.00549 u

Mass of electron antineutrino = 0

What is energy given out by the above decay in MeV? What form does it take?

12.07.5

A thermal neutron is one that has a kinetic energy of about 1 eV. How fast does a thermal neutron travel?

12.07.6

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n + \text{energy}$$

Data to use:

Mass of deuterium nucleus = 3.3425×10^{-27} kg

Mass of tritium nucleus = 6.6425×10^{-27} kg

Mass of helium nucleus = 6.6465×10^{-27} kg

Mass of a neutron = 1.675×10^{-27} kg

What is the energy in J and eV released in this reaction above?

Tutorial 12.08 Nuclear Power	
All Syllabi	
Contents	
12.081 Induced Fission	12.082 Nuclear Power Station
12.083 Safety Aspects	12.084 Transmutation

12.081 Induced Fission

We saw in the last tutorial that fission rarely occurs spontaneously.

We also saw that fission occurs if we "tickle" large nuclei with slow, or **thermal** neutrons. A thermal neutron means that the kinetic energy is equivalent to the photon energy of infra red radiation. A simple kinetic energy calculation shows the speed of the neutrons as being about 14 km s^{-1}, pretty fast for us, but a snail's pace for particles. The electrons in a cathode ray tube of a TV set travel at 5×10^6 m s^{-1}, while particles in high energy physics experiments travel at nearly the speed of light.

We also saw in the last topic that the fission of Uranium nuclei results in a chain reaction. Although the fission products are not easily predictable, the key point to remember is that **three more neutrons** are produced. These go on to tickle three other uranium nuclei, which each produce three thermal neutrons. As we saw, the energy released in an uncontrolled chain reaction results in a violent explosion.

There is a minimum mass of uranium (or other **fissile material**) before a chain reaction can happen, called the **critical mass**. This is because neutrons can escape before they interact with nuclei. The size of the lump of uranium is about the size of a grapefruit, with a mass of 13 kilograms.

12.082 Nuclear Power Station

The nuclear power station is identical in most respects to a normal power station in that steam is used to turn the turbines, which drive the generators. The difference is in the boiler that produces the steam, the **reactor**.

The uranium is fed to the reactor inside **fuel rods**. These are canisters of stainless steel which have fins to transfer the heat

The reactor harnesses the heat energy produced when the uranium nuclei split. It also controls the reaction so that two out of the three neutrons produced are absorbed. Only one neutron out of the three goes on to tickle another nucleus. If any more neutrons are produced, the reaction would start to go out of control. If fewer are produced, the reaction stops. This is achieved by:

- **Moderator**, which slows fast neutrons from the fission to slow thermal neutrons by repeated collisions with the nuclei of the moderator material. Graphite or water are commonly used as moderators.
- **Control rods** made of boron or cadmium. These absorb neutrons. If the control rods are fully in, the neutrons are absorbed completely. At a certain level, the ideal is reached and the reactor is balanced. If the control rods are too far out, then more neutrons than needed can cause the chain reaction to go out of control.

The coolant gas (carbon dioxide, helium) is at high temperature, up to 650 °C and transfers the energy as heat to the heat exchanger. This in turn boils the water to turn the turbines. In a pressurised water reactor (*Figure 41*), liquid water at 320 °C is taken to the heat exchanger.

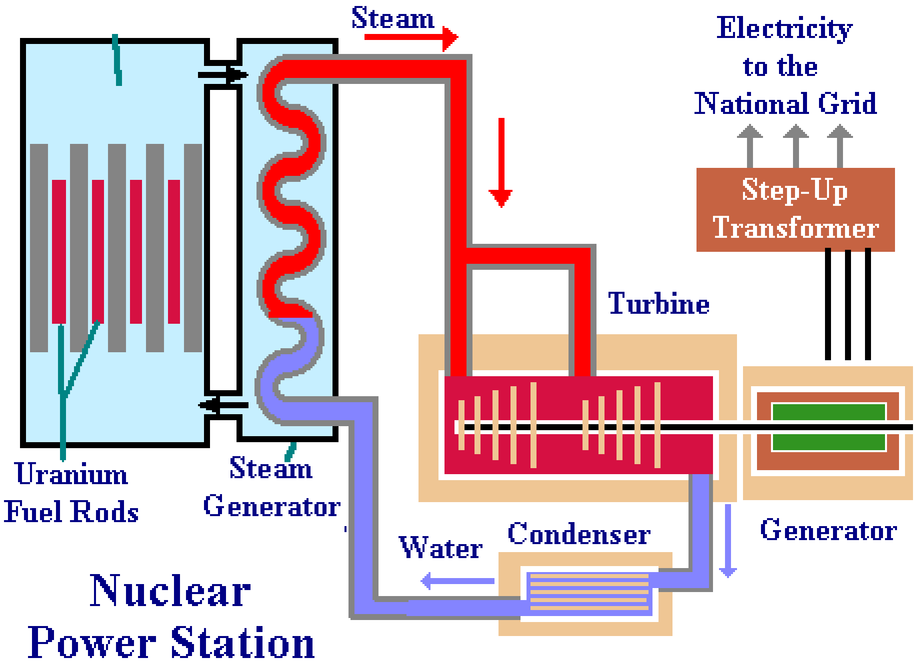


Figure 41 A pressurised water reactor

The reactor is housed in a large steel vessel surrounded by several metres of concrete to stop the radiation from getting out.

The table below shows the different sorts of materials used in different kinds of reactor.

Reactor Type	***Fuel***	***Coolant***	***Moderator***
Magnox (gas cooled)	Uranium encased in a magnesium alloy can	CO_2 at 400 °C	Graphite
AGR (advanced gas cooled reactor)	Uranium dioxide in a stainless steel can	Helium at 650 °C	Graphite
PWR (pressurised water reactor)	Uranium dioxide pellets in a zirconium can	Water under pressure at 320 °C	Water

The type of reactor built depends on many factors, not least the cost. Nuclear power stations have to have many built-in safety systems, as a result of which they are very expensive to build and run. They also have a limited lifespan. The intense radiation produced can weaken the reactor vessel. To replace the vessel requires decommissioning, a long and highly expensive process.

Nuclear Power stations have the advantage that:

- They are very clean, producing no polluting gases;
- They need a lot less fuel. 1 kg uranium can give as much energy as 25 tonnes of coal.

The disadvantages are:

- Expensive to build and run
- Produce dangerous waste.

France generates 80 % of its power with nuclear power stations. Its last coal mine closed in 2004. Britain generates 20 % using nuclear. They remain extremely controversial and inextricably linked with the production of nuclear weapons.

12.083 Safety Aspects

The hazards associated with the nuclear power generation industry are well known and were shown in sharp focus on Saturday 26th April 1986. An unauthorised experiment was carried out at the nuclear power station at **Chernobyl** in which the operators overrode safety systems to enact a worst case scenario failure. They found out. The reactor became unbalanced, and went out of control. The overheating caused decomposition of water into hydrogen and oxygen and these gases collected at the top of the vessel. Mixed with carbon monoxide from the graphite core, the mixture ignited in a thunderclap explosion, which blew the lid off the reactor and turned the vessel on its side.

The damage was done by a chemical explosion, not nuclear. However many tonnes of radioactive muck was hurled into the air, and nine tonnes of caesium-137 floated across Europe. Catastrophic environmental damage was done in the local environment and 135 000 people were evacuated permanently.

The then Soviet authorities tried desperately to cover up the accident, claiming that the accident was a fire in a limestone works. Eventually they had to come clean, and ask for international help to clear up the mess.

The area around the power station is heavily contaminated, and has been abandoned. Ironically, as it has become wild again, the eco-system had thrived and the area has become a haven for a number of rare species that have adopted it as their new home. (They clearly cannot read the notices that say "Radioactive area. Keep out".)

A more recent accident that was just as severe took place on Friday 11th March 2011. The North East of Japan was rocked by a severe earthquake, which was accompanied by a catastrophic tsunami. The nuclear power station at **Fukushima Dai-ichi** (Fukushima Number 1) had its electricity supply interrupted. Its reactors shut down as they were supposed to, and diesel generators cut in to keep power to the plant. Unfortunately these were on the shore-line and were swamped by the waves from the tsunami, and wrecked. The plant went on to battery-power, but then the batteries went flat, leaving the reactors to overheat. Despite the heroic efforts of the staff, each one of the four reactors in turn blew up. The resulting mess will take many years to clear up.

Another less serious but high profile case happened at **Three Mile Island** in the United States of America, blamed on incompetence and corporate failure.

The safety of nuclear facilities has to be of paramount importance, and many systems are built in to prevent failure. The last resort is to drop the control rods into the reactor.

In normal operation, nuclear power generation is very safe; there have been few accidents involving radiation to personnel, although there are the "normal" industrial accidents that happen from time to time. It is right and proper that there are strict controls, for the waste from nuclear reactors is some of the nastiest muck known to man, with radioactive isotopes with long half-lives. Britain processes nuclear waste, a valuable economic business which has to be monitored very carefully. However the reputation of the industry was dealt a major blow some years ago when there was a serious breach of trust by employees at Sellafield who falsified documentation about batches of waste.

The disposal of waste has to be done with considerable care, and remains a truly controversial issue.

12.084 Transmutation

For many centuries **alchemists** tried very hard to make gold by mixing various substances together. They did not have a snowball's chance in Hell of doing so.

However the work of alchemists did give rise to the discipline of chemistry.

To alter elements at the nuclear level, we need to carry out a process of **transmutation**, whereby one element can be turned into another:

- by natural processes, i.e. radioactive decay;
- by bombarding the element with particles that are fast enough to penetrate the nucleus.

Transmutation will occur in the particle bear-garden of a reactor. The first **artificial transmutatio**n was carried out by Rutherford in 1919, converting nitrogen to oxygen with alpha particles:

$$^{14}_{7}\mathrm{N} + {}^{4}_{2}\alpha \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$$

.... *Equation 68*

If we measure the mass off the products carefully, we see that it is greater than the combined masses of the nitrogen nucleus and the alpha particle. **Kinetic energy** from the alpha particle has been **converted** to this mass. This is not as strange as it may seem; at this level mass and energy are interchangeable.

Transmutation is put to good use in a modern form of alchemy, which is the production of radioisotopes that are used in medicine. These need to be of short half-life because:

- the radiation could damage the patient;
- the patient would pose a risk of exposing others to radiation.

A typical transmutation is that of Tellurium-130 being bombarded with neutrons to form Tellurium-131 which decays by beta minus decay to Iodine-131:

$$^{130}_{52}\mathrm{Te} + {}^{0}_{1}\mathrm{n} \rightarrow {}^{131}_{52}\mathrm{Te} \rightarrow {}^{131}_{53}\mathrm{I} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}_{\varepsilon}$$

.... *Equation 69*

Radioactive nuclides such as iodine-131 are used as tracers. A camera sensitive to gamma rays is placed next to the thyroid of a patient and can monitor the uptake of iodine in the patient's thyroid gland.

A further discussion on fission and fusion can be found in the previous tutorial.

Tutorial 12.08 Questions

12.08.1

What kind of nucleus undergoes fission?

12.08.2

Why do the neutrons need to be slow?

12.08.3

Why did alchemists have no chance of producing Gold?

12.08.4

Why do the particles have to travel at a certain speed?

12.08.5

Where does the energy for this transmutation come from?

Answers to Questions

Tutorial 12.01

12.01.1

Changing one nucleus of a different element to another…

…By loss or gain of a particle.

12.01.2

Radiation	***Particle***	***Range in air***	***Stopped by***
Alpha	Helium nucleus	Few mm	Paper
Beta	High speed electron	Few cm	Aluminium sheet
Gamma	Energetic photon	Infinite	Several cm lead

12.01.3

Alpha is intensely ionising.

Although short range and kept out by skin

Ingestion of alpha emitters can do immense damage to the cells.

Beta is less ionising

But can penetrate the body.

Gamma is highly penetrating

But causes little ionisation.

Long term exposure leads to damage to DNA

12.01.4

Both particles lose all their energy in 4.8×10^{-13} J ÷ 5×10^{-18} J

= 96 000 collisions.

The alpha particle has a range of 96000 ÷ 10^5 cm^{-1} = **0.96 cm**

The beta particle has a range of 96000 ÷ 1000 cm^{-1} = **96 cm**.

12.01.5

(a)

Energy of the particle = 5.45×10^6 eV × 1.60×10^{-19} J eV^{-1} = 8.73×10^{-13} J

Speed of alpha particle:

v^2 = (2 × 8.73×10^{-13} J) ÷ 6.64×10^{-27} kg = 2.627×10^{14} m^2 s^{-2}

v = (2.627×10^{14} m^2 s^{-2})$^{0.5}$ = **1.62×10^7 m s^{-1}**

(b) Radius:

$r = mv/Bq$ = (6.64×10^{-27} kg × 1.62×10^7 m s^{-1}) ÷ (0.86 T × 2 × 1.60×10^{-19} C)

r = 0.7821 m = **0.78 m** (2 s.f.)

(c)

Use Fleming's Left Hand Rule:

The direction is upwards.

Tutorial 12.02

12.02.1

Alpha particles could pass through atoms

without deflection

so mostly empty space

Alpha particle is positively charged

Deflection is due to repulsion from positively charged nucleus

12.02.2

(a)

Charge on the alpha particle = $2 \times 1.6 \times 10^{-19}$ C = **3.2×10^{-19} C**

(b)

Charge on the gold nucleus = $79 \times 1.6 \times 10^{-19}$ C = **1.26×10^{-17} C**

(c)

Energy of the alpha particle = 5 MeV = 5×10^{6} eV × 1.6×10^{-19} J eV^{-1} = **8.0×10^{-13} J**

(d)

Closest approach,

$$r = 9.0 \times 10^{9}\ \text{m F}^{-1} \times \frac{2 \times 1.6 \times 10^{-19}\ \text{C} \times 79 \times 1.6 \times 10^{-19}\ \text{C}}{8.0 \times 10^{-13}\ \text{J}} = \mathbf{4.6 \times 10^{-14}\ m}$$

Tutorial 12.03

12.03.1

Yes.

The number of nucleons goes down by four

The number of protons goes down by two.

12.03.2

(a)

$$^{19}_{8}O \rightarrow {}^{19}_{9}F + {}^{0}_{-1}e^{-} + {}^{0}_{0}\overline{\nu}_{e}$$

(b)

$$^{212}_{84}Po \rightarrow {}^{208}_{82}Pb + {}^{4}_{2}He + energy$$

(c)

$$^{56}_{27}Co \rightarrow {}^{56}_{26}Fe + {}^{0}_{1}e^{+} + \nu_{e}$$

12.03.3

Yes

The charge on the LHS = Charge on RHS

Q: +1 → 0 + +1 + 0

12.03.4

Formed when a nucleus is excited.

by a particle being given off in a decay.

The excess energy caused by the change in mass defect

is converted into a photon of the same size as the nucleus.

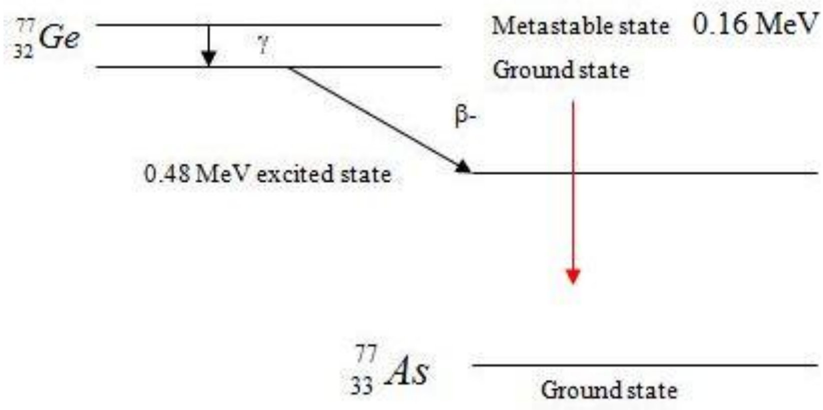
12.03.5

Change in energy = 1.02 - 0.83 = 0.19 MeV = 0.19×10^{6} eV

Change in energy = $0.19 \times 10^{6} \times 1.6 \times 10^{-19} = 3.04 \times 10^{-14}$ J

$\lambda = hc \div E = (6.63 \times 10^{-34} \times 3 \times 10^{8}) \div 3.04 \times 10^{-14} =$ **6.5×10^{-12} m**

12.03.6



(b)

$E = 0.16 \times 10^{6} \times 1.6 \times 10^{-19} = 2.56 \times 10^{-14}$ J

$\lambda = hc \div E = (6.63 \times 10^{-34} \times 3 \times 10^{8}) \div 2.56 \times 10^{-14} =$ **7.8×10^{-12} m**

(c)

There are three possible modes:

0.48 MeV.

0.27 MeV.

0.21 MeV (= 0.48 - 0.27)

1 MeV = 1.6×10^{-13} J

Energy of 0.48 MeV photon = $0.48 \times 1.6 \times 10^{-13}$ = **7.68×10^{-14} J**

Energy of 0.27 MeV photon = $0.27 \times 1.6 \times 10^{-13}$ = **4.32×10^{-14} J**

Energy of 0.21 MeV photon = $0.21 \times 1.6 \times 10^{-13}$ = **3.36×10^{-14} J**

Tutorial 12.04

12.04.1

Corrected count rate = 2550 – 50 = 2500 min^{-1}

Corrected count rate at new position = 6000 – 50 = 5950 min^{-1}

Formula: $I = kI_0/x^2$

2500 min^{-1} = $kI_0/(0.20\ m)^2$ and 5950 min^{-1} = kI_0/x^2

k is the same in both, and I_0 is the same.

Rearranging:

2500 min^{-1} × 0.04 m^2 = kI_0 = 5950 min^{-1} × x^2

$$x^2 = \frac{2500\ min^{-1} \times 0.04\ m^2}{5950\ min^{-1}} = 0.0168\ m^2$$

x = **0.13 m**

Tutorial 12.05

12.05.1

$$\text{No of particles} = \frac{1000}{14} \times 6 \times 10^{23} \times 1.4 \times 10^{-12} = 6 \times 10^{13}$$

Use $dN/dt = -\lambda N$

$-260 = -\lambda \times 6 \times 10^{13}$

$\lambda =$ **4.3×20^{-12} s^{-1}**

12.05.2

Formula:

$$N = N_0 e^{-\lambda t}$$

$$½ = e^{-1.36 \times 10\text{^}-11 \times t1/2}$$

$$\ln ½ = -1.36 \times 10^{-11} \times t_{1/2}$$

$$t_{1/2} = -0.693 \div -1.36 \times 10^{-11} = 5.1 \times 10^{11} \text{ s (= 1600 years)}$$

(Note: 10^11 means 10^{11}. I cannot do a double superscript.)

12.05.3

We need to know how many half-lives gives 5 %

$$(0.5)^y = 0.05$$

$$y \times \log_{10}(0.5) = \log_{10}(0.05)$$

$$y \times -0.3010 = -1.3010$$

$y = -1.3010 \div -0.3010 = 4.32$ half lives

Time taken = 4.32×28 years

Time taken = **121 years**

You can use natural logarithms:

$$(0.5)^{y} = 0.05$$

$$y \times \ln(0.5) = \ln(0.05)$$

$$y \times -0.6931 = -2.996$$

$$y = -2.996 \div 0.6931 = 4.32 \text{ half lives}$$

$$\text{Time taken} = 4.32 \times 28 \text{ years}$$

Time taken = **121 years**

12.05.4

(a)

The activity is 334 $s^{-1} \div 0.1$ = **3340 s^{-1}**

(b)

$$dN/dt = -\lambda N$$

$-3340\ s^{-1} = -\lambda \times 1.5 \times 10^{9}$ = **$2.23 \times 10^{-6}\ s^{-1}$**

(c)

$$N = N_0\, e^{-\lambda t}$$

$$\tfrac{1}{2} = e^{-2.23 \times 10\text{^}-6 \times t}$$

$$\ln \tfrac{1}{2} = -2.23 \times 10^{-6}\ s^{-1} \times t$$

$$t = -0.693 \div -2.23 \times 10^{-6}\ s^{-1} = 3.1 \times 10^{5}\ s$$

t = **3.6 days**

12.05.5

Work out the decay constant:

$$\lambda = 0.693 \div (1250 \times 10^{6}\ \text{y} \times 365\ \text{dy} \times 86400\text{s}) = 1.76 \times 10^{-17}\ \text{s}^{-1}$$

$$-\lambda t = \ln N - \ln N_0$$

$$-1.76 \times 10^{-17}\ \text{s}^{-1} \times t = \ln (500 \times 10^{6}) - \ln (608 \times 10^{6})$$

$$-1.76 \times 10^{-17}\ \text{s}^{-1} \times t = -0.196$$

t = **1.1×10^{16} s** (350 million years)

Tutorial 12.06

12.06.1

$$F = 9 \times 10^{9}\ \text{m F}^{-1} \times \frac{Q1 \times Q2}{r^2} = \frac{9 \times 10^{9}\ \text{m F}^{-1} \times 1.6 \times 10^{-19}\ \text{C} \times 1.6 \times 10^{-19}\ \text{C}}{(1 \times 10^{-15}\ \text{m})^2}$$

F = **230 N**

On a nuclear scale this is a pretty large force.

12.06.2

It must be at least 230 N

to balance out

the repulsive force of the positive charge.

12.06.3

Nucleus is very small (about 10^{-14} m)

and positively charged

Electrons orbit around the outside

In between is empty space.

12.06.4

$R = 1.4 \times 10^{-15}\ \text{m} \times \sqrt[3]{56}$

$R = 1.4 \times 10^{-15}\ \text{m} \times 3.83$

R = **5.36×10^{-15} m**

There would be no difference because there are the same number of nucleons.

12.06.5

$R = 5.36 \times 10^{-15}$ m

$V = 4/3 \times \pi \times (5.36 \times 10^{-15}\ \text{m})^3 = 6.45 \times 10^{-45}\ \text{m}^3$ (not very big)

Mass = 1.67×10^{-27} kg × 56 = 9.35×10^{-26} kg

Density = mass ÷ volume = 9.35×10^{-26} kg ÷ $6.45 \times 10^{-45}\ \text{m}^3$

= **$1.45 \times 10^{18}\ \text{kg m}^{-3}$** (which is quite a lot)

Tutorial 12.07

12.07.1

There are 2 electrons as the proton number for helium is 2.

We need to take away the mass of two electrons

Nuclear mass = 3.016030 u – (2 × 0.000549 u) = **3.014932 u**

12.07.2

m = 0.030377 u × 1.661 × 10^{-27} kg = 5.046 × 10^{-27} kg

$E = mc^2$ = 5.046 × 10^{-27} kg × (3.0 × 10^8 m s^{-1})^2 = **4.541 × 10^{-12} J**

12.07.3

Number of protons = 29; number of neutrons = 63 – 29 = 34.

Mass of protons = 29 × 1.007276 = 29.211004 u

Mass of neutrons = 34 × 1.008665 = 34.29461 u

Total mass = 29.211004 u + 34.29461 u = 63.505614 u

Mass defect = 63.505614 u – 62.91367 u = **0.591944 u**

Mass defect in kg = 0.591944 u × 1.661 × 10^{-27} kg = **9.83218 × 10^{-28} kg**

Binding energy = mc^2 = 9.83218 × 10^{-28} kg × (3 × 10^8 m s^{-1})^2 = **8.85 × 10^{-11} J**

Binding energy in eV = 8.85 × 10^{-11} J ÷ 1.6 × 10^{-19} J eV^{-1} = **5.53 × 10^8 eV**

Binding energy per nucleon = 5.53 × 10^8 eV ÷ 63 = **8.78 × 10^6 eV**

12.07.4

Mass on right hand side = 28.96880 u + 0.000549 u + 0 = 28.969349 u

Mass defect = 28.97330 u - 28.969349 u = 0.0039510 u

Energy = 931.5 MeV u^{-1} × 0.0039510 u = 3.68 MeV

Energy is kinetic.

12.07.5

Kinetic energy = 1 eV = 1.6×10^{-19} J

Mass of a neutron = 1.008665 u × 1.661×10^{-27} kg = 1.675×10^{-27} kg

$$v^2 = 2E_k/m = \frac{2 \times 1.6 \times 10^{-19}\ \text{J}}{1.675 \times 10^{-27}\ \text{kg}} = 1.91 \times 10^{8}\ \text{m}^2\ \text{s}^{-2}$$

v = 13 800 m s^{-1}

12.07.6

Mass on the left hand side = 3.3425×10^{-27} kg + 6.6425×10^{-27} kg = 9.985×10^{-27} kg

Mass on right hand side = 6.6465×10^{-27} kg + 1.675×10^{-27} kg = 8.3215×10^{-27} kg

Mass deficit = 9.985×10^{-27} kg - 8.3215×10^{-27} kg = 1.6635×10^{-27} kg

Energy = 1.6635×10^{-27} kg × $(3.0 \times 10^{8}\ \text{m s}^{-1})^2$ = **1.50×10^{-11} J**

Energy in eV = 1.50×10^{-11} J ÷ 1.6×10^{-19} J eV^{-1} = **9.4×10^{8} eV** = 940 MeV

Tutorial 12.08

12.08.1

A large nucleus

Of which the diameter is about the size of the range of the strong force.

12.08.2

If they are too fast, they will go through the nucleus

And get beyond the attractive region of the strong nuclear force.

12.08.3

Chemical reactions occur at the electron level.

To have any hope of building gold atoms, you have to work at the nuclear level.

To fuse nuclei, you need massively high temperatures

Such as those in a supernova explosion.

12.08.4

If they are too slow, the strong nuclear force will repel them.

If they are too fast, they will go straight through the nucleus.

12.08.5

The kinetic energy of the neutron.